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An Analysis of Pre-Service Elementary Teachers' Understanding of Mathematical Number Sense as Tested on the TIMSS Assessment

Monica J. Doriney
Kennesaw State University

An Analysis of Pre-Service Elementary Teachers' Understanding of Mathematical Number Sense as Tested on the TIMSS Assessment

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Monica J. Doriney

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Abstract

This study investigated pre-service elementary teacher's performance on released items from the 2011 Trends in International Mathematics and Science Study (TIMSS) 4th grade mathematics assessment. This study combined an error analysis of pre-service teacher's errors with a chi-square test for association between type of error and question type as well as type of error and type of cognitive domain. Only number sense questions were chosen, and all questions included were rewritten to (1) be in free response form and (2) encourage pre-service teachers to show all of their work when answering the questions. The test was administered over a two-month period and an analysis of student's results was conducted after all assessments were complete.

The error analysis indicated that these pre-service teachers made the following types of errors: number selection, missing step, computation, operation, random, and omission. Furthermore, the error analysis showed that the pre-service teachers who attempted the questions made mostly missing step and computation errors. A chi-square test for association was also used to determine whether a relationship existed between type of error and cognitive domain (knowledge, applying and reasoning) and between type of error and question type (whole number, patterns and relationship, number sentences with whole numbers, and fractions and decimals). The test produced statistically significant results between error type and cognitive domain, indicating a relationship between error type and cognitive domain. However, the chi-square test did not indicate a relationship between error type and content domain. Thus, the results suggest that the types of errors committed are similar for each question type.

INDEX WORDS: Mathematical content knowledge, Error Analysis, Pre-service Elementary Teachers, TIMSS Assessment

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Chapter 1: Introduction and Rationale

Statement of Problem

In recent years, there have been many international, national and state assessments that show that the United States is falling behind comparable countries in mathematics achievement (NCES, 2012). Specifically, international tests, used to compare countries academically, show that the U.S. is not decreasing in academic achievement, but is, however, not increasing at the rate of other comparable countries (Hanushek et al, 2012). The need to increase student achievement in mathematics is evident, as the United States needs to continue to be a world contender in education (Hanushek et al, 2012). Educational researchers (e.g., Benner & Hatch, 2009) have determined that the main way to increase student achievement is to properly prepare teachers for effective teaching (Benner & Hatch, 2009).

A number of researchers (e.g., Ball, 1990; Lange & Meaney 2011; Ryan & Williams, 2007) have expressed concern regarding pre-service teachers' understanding of mathematics. Much of the research on pre-service teachers' understanding of mathematics (e.g., Ball & Bass, 2000; Ball, Lubienski & Mewborn, 2001; Hill, Ball & Schilling, 2008) has shown that effective teachers need both an understanding of students' mathematical thinking as well as an in-depth understanding of mathematical content. Shulman (1986) proposed categories of knowledge necessary for effective teaching (Figure 1). Of most relevance for this study, Shulman's (1986) *content knowledge* requires a deep understanding of how concepts, problems and issues are organized and the teacher's ability to adapt instruction in their discipline to the needs, interests and abilities of the students.

Figure 1
Shulman (1986) Categories of the Knowledge Base

- General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter
- Knowledge of learners and their characteristics
- Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures
- Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds
- Content knowledge
- Curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade " for teachers
- Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding

Several studies, based on Shulman's (1986) work, have investigated how teachers gain mathematical knowledge as well as how they apply it when teaching (e.g., Ball, 1990; Borko, 1992; Stein, 1990). Using the work of Shulman (1986) regarding pedagogical content knowledge, Ball, Hill and Schilling (2008) categorized the mathematical knowledge teachers need to be effective as *mathematical knowledge for teaching* (MKT). These researchers (Hill et al., 2008) discussed pedagogical content knowledge (PCK), which is combining pedagogical knowledge and subject matter knowledge. They (Hill et al., 2008) also defined mathematical knowledge for teaching as a deepened understanding of the teaching mathematics that includes ways of representing it, explaining it, and modeling it. Additionally, these researchers (Hill et al., 2007) defined subject matter knowledge as both the mathematical knowledge that is held by well-educated adults and the mathematical knowledge learned in school. In addition, they (Hill et al., 2008) found pedagogical content knowledge to be the knowledge of what concepts students struggle with mathematically and knowledge of what mistakes and misconceptions students may have, along with how to adapt mathematics instruction effectively for the needs of learners. Due to the significance of teacher knowledge, researchers (Ball, 2000; Ma, 1999) found that teachers need to have an understanding of the mathematics that they are required to teach together with the ability to explain various mathematical concepts in detail. However, researchers (e.g., Ball, 2000; Ryan & Williams, 2007) have established that elementary pre-service teachers experience some of the same difficulties with fundamental mathematical concepts and skills as the students they teach such as algebraic concepts and basic number sense.

Today the Common Core State Standards (CCSS) and increased demands for accountability are forcing elementary and middle school teachers to have a deeper understanding of the foundations of algebra so that they can impart their knowledge to their students (NCTM, 2010; Hill, Rowan, & Ball, 2005; Ma, 1999). Therefore, it is critically important for pre-service elementary school teachers to achieve a deep understanding of mathematical knowledge for teaching prior to becoming teachers (Ball, 2000; Ma, 1999).

Purpose and Significance of the Study

The purpose of this study is to investigate in-depth the various types and frequency of errors made by elementary pre-service teachers on number sense, the key mathematics topic in the elementary curriculum. The goal of this study is to investigate elementary pre-service teachers' types of errors made on number sense problems with the goal of analyzing their mathematical knowledge and their readiness to teach. Therefore, the research questions for this study are

1) What types of errors are made by elementary pre-service teachers in mathematics?

2) What is the frequency of these errors?

3) Is there a relationship between types of errors and cognitive domain?

4) Is there a relationship between types of error and question type?

An error analysis of teacher candidate work on released problems from the Trends in International Mathematics and Science Study (TIMSS- an international assessment), along with a comparison of results from the scores for U.S. fourth grade students will provide some insight on how well prepared elementary school pre-service teachers are in their mathematical content knowledge. This analysis will add to our understanding of information on how well these typical pre-service teachers know the number sense concepts that they will teach.

Review of Relevant Terms

Computation Error: An error was made in the calculation of the problem (Meyer, 1985).

Content Knowledge: Knowledge of the subject matter being learned or taught (Shulman, 1986).

***Curriculum Knowledge**:* Knowledge of how curriculum functions to engage student in a particular context (Shulman, 1986).

Error Analysis: The analysis of error patterns to identify difficulties that students may have with facts, concepts, strategies and procedures.

General Pedagogical Knowledge: Knowledge about the methods of teaching and learning (Shulman, 1986).

Mathematical Knowledge for Teaching: The mathematical knowledge needed for teaching mathematics to students (Ball, Hoover, Thames, & Phelps, 2008)

Missing Step Error: An error made from completing fewer steps than needed to solve the problem (Meyer, 1985).

Number Selection Error: An error caused from using the wrong number or putting it in the wrong place (Meyer, 1985).

Number Sense: Number sense is an intuition about numbers that is drawn from all the varied meanings of the number (NCTM, 1989).

Omission Error: The entire question is left blank (Meyer, 1985).

Operation Error: An error caused from using the wrong operation to solve the problem (Meyer, 1985).

Pedagogical Content Knowledge: Knowledge of the ways of representing and explaining the subject (Shulman, 1986).

Pre-Service Teacher: The elementary education majors in this study were in their final years of their undergraduate degrees and had completed all or all but one of their mathematics courses. They are not yet certified to teach, but are scheduled to graduate and begin teaching within 12-18 months.

Random Error: An error was made with no justification (Meyer, 1985).

Chapter 2: Review of Literature

Langham, Sundberg, and Goodman (2006) discussed the issue that teachers cannot teach mathematics effectively without a deep understanding of the curriculum. Gadanidis & Namukasa (2007) also assert that beginning teachers need to have a strong foundation and a well-connected understanding of various mathematics concepts within the curriculum in order to be fully prepared to teach mathematics. These findings (Gadanidis & Namukasa, 2007) indicate that elementary education teachers need to have at deep understanding of at least elementary school mathematics. Other researchers (Hill, Sleep, Lewis, & Ball, 2007; Philipp et al., 2007; Stylianides & Ball, 2008) extended those findings and elaborated on the concept of what mathematical understanding teachers need. Although these researchers argue that the depth of teacher knowledge needs to go beyond what is taught in the curriculum, the extent of this depth is not well defined (Ball, Bass, & Hill, 2004). This lack of an adequate definition presents a problem for determining what concepts and skills need to be taught to pre-service teachers beyond that in the curricula they teach (Ball, Bass, & Hill, 2004).

Theoretical Framework

Introduction. Research on teacher knowledge has increased greatly over the last two decades (Peng & Luo, 2009). Most of this development is directly related to the work done by Shulman in 1986. Shulman's (1986) work not only discussed the need for subject matter knowledge and curricular knowledge, but also the need for pedagogical content knowledge (PCK). His work motivated many studies (Adler & Davis, 2006; Ball, Hill & Bass, 2005; Peng, 2007; Even & Tirosh, 1995) that focused on the need for specialized content knowledge that teachers need to teach effectively, as well as the specific knowledge that teachers need to teach specific content areas. Additionally, a few studies (Peng & Luo, 2009) focused on mathematical errors of students and the knowledge teachers need in order to correctly analyze and address those errors. However, there have been few studies that focused on the errors that teachers make and how those mathematical errors demonstrate limitations to teacher knowledge (Peng & Luo, 2009).

The value of analyzing student's mathematical errors has been recognized as useful for many years (Radatz, 1979). Radatz (1979) used error analysis to analyze students' errors and classify them into categories based on student behavior. This was done using a cognitive information processing model to analyze student understanding. Radatz (1979) classified errors in terms of language difficulties, processing iconic and visual representation difficulties, association difficulties, and application difficulties. He (Radatz, 1979) found that students had difficulties understanding mathematical language, demonstrating mathematical knowledge, and recalling, transferring, and decoding information.

The main purpose of error analysis is to develop a model of students' misconceptions from studying the types of errors that the student committed (Brown & Burton, 1978). Although there have been many studies regarding error analysis with students, very little research has been done using error analysis on the knowledge of mathematics teachers (Peng & Luo, 2009). In one related study, Tunuklu and Yesildere (2007) conducted a study on teacher knowledge using error analysis based on students' errors, not the teachers' errors. Teachers in their study were asked to analyze student errors on various levels of mathematics. By studying the analysis done by these teachers, Tunuklu and Yesildere (2007) were able to draw conclusions about the specific knowledge of not just the students, but also the teachers who performed the error analysis. Turnuklu and Yesildere (2007) found that the teachers in their study had a sound understanding of elementary mathematics, but did not have the necessary knowledge required for teaching mathematics. They (Turnuklu & Yesildere, 2007) argued that primary mathematic candidates need to be taught both mathematics knowledge and pedagogical content knowledge. This would bridge the necessary connection between mathematical knowledge and the knowledge needed to teach mathematics (Turnuklu & Yesildere, 2007).

Additionally, Moru and Qhobela (2013) conducted a similar study in which teachers were asked to analyze student errors in attempt to assess teacher knowledge. Moru and Qhobela (2013) found that a teacher's ability to identify the errors of the content was related to the teacher's knowledge of that mathematical concept. Although these studies were beneficial in assessing teacher knowledge through analyzing student errors, none of these studies analyzed the errors actually committed by the teachers (Peng & Luo, 2009). Thus, an error analysis of pre-service teachers would be a useful addition to the literature on teacher knowledge and pre-service preparation to teach mathematics (Peng & Luo, 2009).

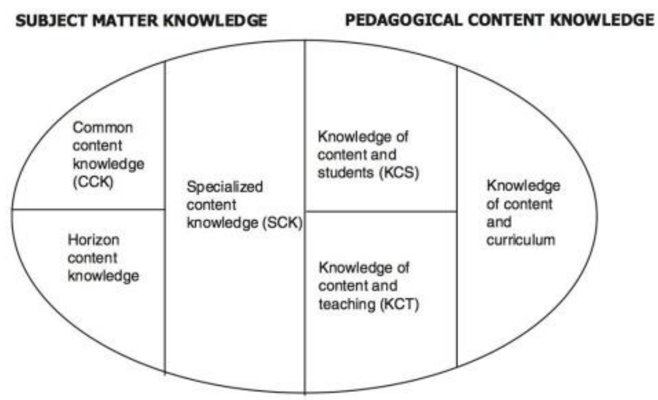
Teacher Knowledge Framework. The perspective used to frame this study involving elementary pre-service teachers' understanding of number is based on Shulman's (1986) construct of teacher knowledge. Shulman's (1986) construct includes content knowledge, general pedagogical knowledge, curriculum knowledge, and pedagogical content knowledge (Figure 2). According to Shulman (1986), pedagogical content knowledge is the specialized knowledge that teachers need in order to be effective for teaching a specific subject. Shulman (1987) described this knowledge as

> The blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for inspection. Pedagogical content knowledge is the category of knowledge most likely to distinguish the understanding of the content specialist from that of the pedagogue (Shulman, 1987, p. 8).

This study assessed content knowledge, which is a necessary foundation for PCK and for mathematical knowledge of teaching (MKT), as presented by Ball, Bass, and colleagues (e.g., Hill et al, 2008). MKT is a construct that employs both subject matter knowledge and pedagogical content knowledge (Hill et al, 2008). Although both of these types of knowledge are specific to mathematics teaching, pedagogical content knowledge is the knowledge of mathematical pedagogy while subject matter knowledge is the knowledge of mathematics content (e.g., Hill et al, 2008). This study will examine the content knowledge of pre-service teachers to fully understand more about how teachers understand particular mathematics concepts.

Figure 2

Domains of Mathematical Knowledge for Teaching



(Ball, Thames, & Phelps, 2008)

Mathematics Teacher Knowledge Shown Through Error Analysis. Peng & Luo, (2009) noted that the literature does not provide clear understanding on the mathematical knowledge of teachers using error analysis. However, using Shuman's definition of knowledge described above, it seems that an analysis of mathematical error can help to determine whether a teacher has the knowledge needed to be effective. Thus the framework for examining teacher knowledge shown through error analysis is needed to determine teacher misconceptions as well as to formulate a more complete picture of pre-service teacher's knowledge of mathematics (Peng & Luo, 2009). Peng & Luo (2009) explain that being able to determine the reason or the cause of a particular error would fall into an area of specialized content knowledge. Additionally analyzing common errors and predicting what errors students are likely to commit falls into knowledge of the content and knowledge of the student (Peng & Luo, 2009). Understanding why students commit certain errors also involves understanding multiple interpretations of why students make particular errors (Moru & Qhobela, 2013). These multiple interpretations help to determine remediation strategies to address the misconceptions exhibited by the errors committed (Peng & Luo, 2009). Examining teacher knowledge using error analysis also involves analyzing student work through students' explanations of their errors (Moru & Ohobela, 2013). Unfortunately, having students explain their errors is not always feasible, thus teachers need to have the necessary knowledge to determine the possible causes and sources or those errors (Peng & Luo, 2009). Although this research pertains to teachers' understanding of student errors, the same process can be used to understand teacher errors as they relate to content knowledge, general pedagogical knowledge, curriculum knowledge, and pedagogical content knowledge.

Analyzing Pre-Service Teachers Errors. To analyze pre-service teachers' difficulties in solving elementary-level mathematics problems, this study implemented an adaptation of Newman's (1977) Error Analysis (NEA). Newman (1977) created a model to assess student difficulties in solving mathematical word problems, in which he categorized student's errors in five ways: reading errors, comprehension errors, transformation errors, process errors, and encoding errors. These categories directly relate to students' ability to recognize words and symbols, understand the meaning of the problem, translate the word problem to a mathematical expression, perform the correct mathematical procedure, and represent the solution in written form (Wijaya, 2014).

Although Newman's model is adequate in assessing student errors in mathematical word problems, several other researchers modified it to classify student errors or used other approaches. Hodes and Nolting (1998) discussed five types of errors for word problems: reading errors, comprehension errors, transformation errors, procedural errors, and encoding errors. Additionally, Brodie (2005) classified student errors under the perspective that students make errors based on their previous mathematics experiences. These errors can originate in and out of school, and could be classified as expected and appropriate errors (appropriate for the grade level of the child) (Brodie, 2005). Furthermore, Riccomini (2005) discussed errors that seemed to only happen once and errors that happen habitually. Elbrink (2008) classified errors into three major categories: calculation errors, procedural errors, and symbolic errors. Similar to Elbrink (2008), Meyer (1982) created a method for analyzing student errors in any mathematical problems, not solely word problems. Meyer's (1982) theory is based on two major ideas: students' ability to comprehend the problems and students' ability to represent the problems. Meyer (1982) found that mathematical problem solving incorporated different types of knowledge needed to accurately solve the problem. His (Meyer, 1982) types of knowledge involved linguistic and factual knowledge, schema knowledge, algorithmic knowledge, and strategic knowledge. Using Newman's (1977) Error Analysis along with Meyer's (1982) theory and Shulman's (1987) theory construct on teacher knowledge, a six- level classification of errors was adapted for this study (Figure 3).

Figure 3
Types of Errors for Classifying Pre-Service Teacher's Errors (adapted from Newman, 1977)

1. Number Selection Error	The pre-service teacher used the wrong number of put the number in the wrong place when attempting to solve the problem.

2.	Missing Step Error	The pre-service teacher completed the problem in fewer steps than needed to accurately and completely solve the problem.
3.	Operation Error	The pre-service teacher used the wrong mathematical operation to solve the problem.
4.	Computational Error	The pre-service teacher made a mistake in their calculation of the solution to the problem.
5.	Random Error	The pre-service teacher committed an error that could not be classified because the error was committed with no justification.
6.	Omission Error	The pre-service teacher did not answer the question and the question was left blank.

The theories provided by Newman (1977), Meyer (1985), and Shulman (1986) provide the framework for this study exploring the reasons for mathematical errors made by pre-service teachers. Through an analysis of pre-service teachers' errors, teachers' knowledge can be analyzed and assessed using Shulman's classifications of knowledge. In all, this framework will help to identify the errors committed by pre-service teachers and to use that error analysis to assess their knowledge of elementary mathematics.

Assessment of Mathematical Achievement

A number of assessments show that the United States (U.S.) is lagging behind other comparable countries regarding achievement in mathematics (OCED, 2012). International, national, and state tests are used to determine academic achievement, such as the National Assessment of Educational Progress (NAEP) (NAEP, 2013), Trends in International Mathematics and Science Study (TIMSS) (Mullis et al., 2011), Program for International Student Assessment (PISA) (NCES, 2012), Georgia's Criterion-Referenced Competency (CRCT) (GADOE, 2013), Georgia's End-of-Course Assessment (EOC) (GADOE, 2013), and others, are used to determine academic achievement on state, national, and international levels. The results from the various international tests show that although the U.S. is improving annually in mathematics achievement, other countries are making stronger annual gains (Hanushek et al, 2012). The performance of students in mathematics and other subjects is extremely important due to the need for the U.S. citizens to be globally competitive (Hanushek et al, 2012). Although the U.S. did not decrease in academic achievement, other countries around the world are progressing at a much faster rate than the U.S. (Hanushek et al, 2012). For example, students in Latvia, Brazil, and Chile are making academic gains for all tested grade levels at a rate three times faster than the U.S. (Hanushek et al, 2012). Furthermore, countries such as Portugal, Hong Kong, Germany, Poland, and Columbia are making academic gains in mathematics achievement at twice the rate of the U.S. (Hanushek et al, 2012).

The Organization for Economic Cooperation and Development (OECD) was established in 1961 and comprises 18 European counties, the U.S., and Canada (NCES, 2012). The OECD's original purpose was to provide an open and trustworthy way to communicate between the most advanced countries (NCES, 2012). The purpose of the organization has since been redefined to focus on stimulation of high economic growth and expansion of world trade (NCES, 2012). Over the years, OECD has expanded to 34 member countries. One of the OECD's programs is the International Association for the Evaluation of Educational Achievement (IEA), an international organization of national research institutions and governmental research agencies, which conducts the TIMSS (NCES, 2012). The assessment is coordinated by the TIMSS International Study Center at Boston College and focuses on mathematics and science knowledge at grades 4 and 8 (NCES, 2012).

About TIMSS

TIMSS is an international assessment that measures the mathematics and science achievement of 4th and 8th graders (NCES, 2012). This test was created to parallel the common topics in the curricula of the education systems that participate in the assessment (NCES, 2012).

Therefore, the results of the assessment should indicate which concepts and skills students' have mastered in school (NCES, 2012). Additionally, the TIMSS assessment collects data on the background of participating students, teachers, schools, and curriculum (NCES, 2012). This data is used to compare the participating education systems to better analyze student achievement (NCES, 2012). Participation in the TIMSS assessment is open to all countries and education systems within them. In the 2011 TIMSS assessment, a total of 53 education systems internationally participated in the grade 4 assessment, and a total of 57 participated in the grade 8 assessment (NCES, 2012).

Nine states, in addition to the U.S. as a whole, participated in the 2011 TIMSS assessment. Both Florida and North Carolina participated in TIMSS for grades 4 and 8 (NCES, 2012). Alabama, California, Colorado, Connecticut, Indiana, Massachusetts, and Minnesota participated in TIMSS at grade 8 (NCES, 2012). These nine states had public school samples large enough to receive data reports as a separate entity from the U.S (NCES, 2012).

TIMSS assessments were given in 1995, 2003, 2007, and 2011, so TIMSS data is available for the last 16 years (1995 to 2011) and was most recently administered in 2015, although 2015 results are not yet available (NCES, 2012). The student populations sampled to represent the U.S. were randomly selected. In order to make sure that the data was accurate and valid, the sample for each participating education system included at least 4,000 students from at least 150 schools (NCES, 2012). In total, the U.S. national total included 369 schools and 12,569 students for the fourth grade TIMSS assessment, while 501 schools and 10,477 students contributed to the eighth grade TIMSS assessment in 2011 (NCES, 2012). The nine states that participated separately were not included in the sample data provided for the U.S (NCES, 2012).

Reporting of TIMSS Score Results

The purpose of the TIMSS assessment is to compare and contrast student achievement from various countries (NCES, 2012). Therefore individual student scores are not reported. The results on the TIMSS assessment are reported on a scale from 0 to 1,000 (NCES, 2012). Each time the assessment is given, the same scale is used in order to compare student achievement over time (NCES, 2012). The mean score of the assessment is 500 with a standard deviation of 100. The scale, mean, and standard deviation were established in 1995 when the first TIMSS assessment was administered (NCES, 2012). This consistent use of the scale allows countries to compare their own scores from testing to testing as well as their scores to other countries (NCES, 2012).

International benchmarks, in addition to numeric scores, are also used to determine whether students demonstrated the required skills and understanding at each benchmark level (NCES, 2012). The score of 625 is required for advanced, 550 for high, 475 for intermediate, and 400 for low (NCES, 2012). The benchmark percentage indicates the proportion of students who mastered the concepts and skills required for that benchmark level (NCES, 2012). To reach the advanced international benchmark level, students were required to apply their understanding and knowledge in various complex situations (NCES, 2012). To obtain the high international benchmark level, students had to apply their understanding and knowledge to solve problems (NCES, 2012). To obtain the intermediate level, students had to apply basic mathematical knowledge in straightforward situations (NCES, 2012). Students who demonstrated basic mathematical knowledge achieve the lowest international benchmark (NCES, 2012).

Content and Cognitive Domains

The mathematics problems on the TIMSS assessment are categorized in two ways (Mullis et al., 2011). These two categorizations are content domains and cognitive domains (Mullis et al., 2011). Table 1 shows the content domains, including the components of each domain and a description of each component, represented in the mathematics portion of the TIMSS assessment (Mullis et al., 2011). The definitions provided in Figure 4 are based on the framework presented by TIMSS that was outlined in the TIMSS databases for each repetition of the TIMSS test (Mullis et al., 2011). These definitions were established by the International Association for the Evaluation of Educational Achievement (IEA) during the design of the TIMSS assessment (Mullis et al., 2003). Each administration of the TIMSS assessment reports the mean score for each participating country on each content domain (Mullis et al., 2011). Specifically, the content domains for the fourth grade assessment are number, geometry, and data (Mullis et al., 2011).

Figure 4
Definitions of Content Domains Established by IEA (Mullis et. al., 2003)

Mathematics
Number. The number domain consists of whole numbers fractions and decimals, integers, ratio, proportion, and percent. **Geometry.** The geometry domain includes understanding "lines and angles, two- and three-dimensional shapes, congruence and similarity, locations and special relationships, symmetry and transformation." **Data.** The data domain covers "data collection and organization, data representation, data interpretation, and uncertainty and probability." **Algebra.** The algebra domain consists of "patterns and relationships among quantities, using algebraic symbols to represent mathematical situations, and developing

fluency in producing equivalent expressions and solving linear equations."

The cognitive domain components of the TIMSS assessment (Figure 5) consist of knowing facts, procedures, and concepts; applying understanding and knowledge of various concepts; and mathematical reasoning (Mullis et al., 2003). During the design of TIMSS, IEA also created definitions for the various cognitive domains. Mullis (2005) explained that knowledge domain assesses the basic information that students need to know. The applying domain focuses on the students' ability to apply what they know in routine problems and questions (Mullis, 2005), while the reasoning domain assesses the students ability to go well beyond the routine problem solving and assesses their ability to work within unfamiliar situations, difficult contexts and multi-step problems (Mullis, 2005).

Many studies have been conducted that focus on the content domain of the TIMSS assessment (Zonts, 2013). Tatsuoka, Corter, and Tatsuoka (2004) researched the content domains of the 1999 TIMSS assessment. They concluded that the assessment was based on 23 very specific content and processing domains (Tatsuoka et al., 2004). Their research (Tatsuoka et al., 2004) found that U.S students were very strong in some content domains and very weak in others. Specifically, they (Tatsuoka et al., 2004) concluded that U.S students were extremely weak in geometry. While Tatsuoka, Corter, and Tatsuoka (2004) focused on content and processing domains of U.S students, Chen, Gorin, Thompson, and Tatsuoka (2008) analyzed the content domains students from Taiwan. They (Chen et al., 2004) also found that Taiwanese students were very strong in certain content domains and very weak in others. Although these two studies show students' understanding of specific content domains, very few studies focused on student achievement in different cognitive domains (Mullis et al., 2003). Toker (2010) examined the 2007 TIMSS assessment to determine students' strengths and weaknesses of each cognitive domain in Turkey and found that there was no significant difference between the cognitive domains.

Figure 5
Definitions of Cognitive Domains Established by IEA (Mullis et al., 2003)

Mathematics
Knowing. The facts, concepts, and procedures students need to know.
Applying. The ability of students to apply knowledge and conceptual understanding to solve problems or answer questions.
Reasoning. The ability to extend beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multistep problems.

Although the TIMSS report outlines the percentage of correct responses by students for each cognitive domain, the authors (Mullis et al., 2011) of the report do not use the cognitive domains to compare countries. Additionally, while some studies focused on the cognitive domains of the TIMSS assessment, there is only one study that focused on the cognitive domain of the 2011 assessment (Zonts, 2013). This study (Zonts, 2013) used the TIMSS database of the 2011 assessment to analyze the effectiveness of national ranking of international tests using cognitive domains. The researcher (Zonts, 2013) found that the U.S. is falling behind other countries in some cognitive domain areas, but not all. Specifically, Zonts (2013) found that U.S. students struggle with applying and reasoning in science and mathematics.

Content Domain: Number

This study focuses solely on the number content domain (Figure 6). At the fourth grade level, this content domain includes understanding place value, representing numbers in different ways, and understanding the relationship between numbers (Mullis et al., 2011). It is expected that, at the fourth grade level, students should have already developed number sense and conceptual understanding (Mullis et al., 2011). Additionally, fourth-grade students should understand the meaning of different operations and the various ways they can appear in problems (Mullis et al., 2011). Furthermore, students should be able to identify number patterns and explore relationships between numbers (Mullis et al., 2011). Table 3 shows the specific number content domain components exhibited on the TIMSS 2011 fourth-grade assessment (Mullis et al., 2011).

Figure 6

Number Content Domain Components (Mullis et al., 2011)

Mathematics – Fourth-grade assessment

Whole Numbers

- Demonstrate knowledge of place value, including recognizing and writing numbers in expanded form and representing whole numbers using words, diagrams, or symbols.
- Compare and order whole numbers.
- Compute with whole numbers (+, −, ×, ÷) and estimate such computations by approximating the numbers involved.
- Recognize multiples and factors of numbers.
- Solve problems, including those set in real life contexts including those involving measurements, money, and simple proportions.

Fraction and Decimals

- Show understanding of fractions by recognizing fractions as parts of unit wholes, parts of a collection, locations on number lines, and by representing fractions using words, numbers, or models.
- Identify equivalent simple fractions; compare and order simple fractions.
- Add and subtract simple fractions.
- Show understanding of decimal place value including representing decimals using words, numbers, or models
- Add and subtract decimals.
- Solve problems involving simple fractions or decimals

Number Sentences with Whole Numbers

- Find the missing number or operation in a number sentence
- Model simple situations involving unknowns with expressions or number sentences.

Patterns and Relationships

- Extend or find missing terms in a well-defined pattern, describe relationships between adjacent terms in a sequence and between the sequence number of the term and the term.
- Write or select a rule for a relationship given some pairs of whole numbers satisfying the relationship, and generate pairs of whole

	numbers following a given rule (e.g., multiply the first number by 3 and add 2 to get the second number).

Whole numbers are the primary components for operations with numbers (Mullis et al., 2011). The ability to work with whole numbers at the elementary level provides the foundation for the rest of mathematics (Mullis et al., 2011). TIMSS expects that students in fourth-grade should be able to solve basic mathematics problems involving whole numbers of a reasonable size (Mullis et al., 2011). Additionally, these students should be able to use estimation to find sums, differences, products, and quotients, as well as use computation to solve basic mathematics problems (Mullis et al., 2011).

Students should also be able to use number sense to solve and analyze problems involving relationships between measurements and use conversions to change the unit of measurement (Mullis et al., 2011). Specifically students should be able to use multiples of 10 found in the metric system of measurement (Mullis et al., 2011). Also, students should be able to convert time measurements and understand the relationships between seconds, minutes, hours, and days (Mullis et al., 2011).

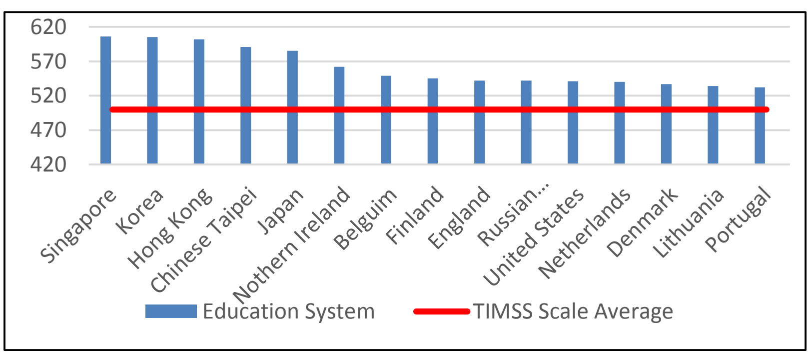
The fourth-grade TIMSS assessment does not include algebraic concepts (Mullis et al., 2011). However, it focuses on the concepts needed for understanding of algebra (Mullis et al., 2011). The number content domain focuses on the type of understanding needed for a deep and conceptual understanding of algebraic thinking (Mullis et al., 2011). Although algebraic concepts are not explicitly included on this assessment, students are still expected to work with number sentences to find missing numbers (Mullis et al., 2011). This idea leads students to learn how to find the value of unknown variables and to use number sentences to solve simple and complex problems (Mullis et al., 2011). Additionally, students should be able to define patterns and explore relationships between terms of a pattern (Mullis et al., 2011). This idea is leads students to a conceptual understanding of functions (Mullis et al., 2011).

TIMSS Results: 2011 administration, Fourth Grade Math

In 2011, the average U.S. fourth grade mathematics scores was lower than international TIMSS average mathematics score at fourth grade for all participants (NCES, 2012). The score for U.S. fourth graders placed the U.S. among the top fifteen education systems in mathematics internationally (Figure 7) (NCES, 2012). On average, the U.S. scored higher than 42 education systems (NCES, 2012). The seven education systems that scored higher on average mathematics scores than the U.S. were Singapore, Korea, Hong Kong, Chinese Taipei, Japan, Northern Ireland, and Belgium (NCES, 2012). Both North Carolina and Florida, which were scored separately from the U.S., scored higher than the TIMSS average score for the U.S. at grade 4 (NCES, 2012). North Carolina scored higher than the U.S. average; however Florida's score was statistically the same as the U.S. national score in mathematics (NCES, 2012).

Figure 7

Top 15 Average Mathematics Scores of Fourth Grade Students, by Educational System 2011 (NCES, 2012)



Between 2007 and 2011, the U.S. fourth grade mathematics score increased 12 points (NCES, 2012). Of all the participating education systems, the U.S. was only one of 12 that showed this increase between the 2007 test administration and the 2011 test administration (NCES, 2012). Additionally, Florida and North Carolina, the two states that independently participated in the TIMSS assessment, scored 545 and 554 on the fourth grade TIMSS assessment in 2011 respectively (NCES, 2012). Although both states scored higher than the U.S. score of 541, the scores of the two states were not measurably different than that of the U.S. as a whole (NCES, 2012).

International Assessments

Compared to the other 34 OECD nations, the U.S. is lagging behind in our rate of improvement in student achievement (NCES, 2012). As shown by the 2011 TIMSS assessments, U.S. 8th and 12th graders are above average in math (NCES, 2012); however, the TIMSS scores over the past three decades have remained constant for the U.S., indicating that the improvements aren't keeping up with what is happening in other countries (Fensterwald, 2013). Countries such as Russia, Vietnam and Germany, which in the past performed at levels below the U.S., have now caught up in ranking to the U.S. (Fensterwald, 2013). TIMSS scores have remained statistically the same despite the introduction or reintroduction of various initiatives with the goal of improving mathematics learning, such as New Math, Back to Basics (Klein, 2003) and more recently, Common Core State Standards. These TIMSS performance results raise the question as to whether changes in curricula or other initiatives over the last three decades have improved the mathematics achievement of elementary school students (Klein, 2003). Many U.S. teachers were educated during these time periods that TIMSS assessments showed U.S. mathematics to lag behind other countries. Do current U.S. pre-service teachers adequately meet the rigorous standards required of teachers to keep the U.S. competitive in the growing global economy?

Teacher Mathematics Knowledge

As mentioned above, Shulman (1986) defined teachers having specialized subject-matter knowledge for teaching as pedagogical content knowledge. Shulman believed that this type of knowledge is unique to teachers and relates what they know about teaching to what they know about what they teach. Leikin (2006) defined the knowledge needed for teachers to be successful as subject-matter knowledge, pedagogical content knowledge, and curriculum knowledge. Moreira and David (2008) believe that the concepts and skills taught in a formal mathematics class and the concepts and skills taught in a teacher preparation class for mathematics are often different, perhaps because of the inclusion of PCK Furthermore, Moreira and David (2008) argue that the techniques and strategies taught in the two settings sometimes conflict. They explain that the way children are taught to solve problems in a formal mathematics class is different from the way future teachers are taught to teach mathematics in a teacher preparation class. Additionally, teaching strategies need to change over time as teachers gain experience and become more effective and as curriculum changes (Chamberlain, 2007). Chamberlain (2007) believes that teachers need to be able to make a smooth transition from learning mathematics content to learning pedagogical strategies used to help students make sense of the mathematics. Ball and colleagues (e.g., Ball & Bass, 2003; Ball, Bass, & Hill, 2004) argues that specialized knowledge of mathematics should be integrated into all teacher preparation programs. Ball defines common mathematics knowledge as the basic skills possessed by a mathematically literate person. In contrast, Ball defines specialized content knowledge as the level of mathematical knowledge required for one to teach mathematics (Ball, 2005). Additionally, Ball (2005) stated that the mathematical knowledge needed for teaching mathematics is not just the mathematics knowledge commonly held by mathematically literate adults (Ball, 2005).

In 2008, the Report of the National Mathematics Advisory Panel stated that teachers need to have detailed knowledge of and an advanced perspective on the mathematical concepts that they teach. The report further argues for the need of specialized content instruction in teacher preparation programs. Additionally, a report published by the National Council on Teacher Quality (2008) also argues that teachers need to possess a deep conceptual understanding of mathematics. This NCTQ report stated that pre-service teachers need to demonstrate a deeper understanding of mathematical concepts than that of the students they plan to teach. Ma (1999) specifically indicates that teachers of elementary mathematics need to be skilled enough to provide students with a deep and firm understanding of various mathematical concepts.

Procedural and Conceptual Knowledge

Procedural knowledge encompasses computational skills, while conceptual knowledge requires a deep understanding of mathematical relationships and a structural understanding of mathematical ideas (Eisenhart, Borko, Underhill, Brown, Jones, & Agard, 1993). The development of deep conceptual knowledge in pre-service teachers is challenging (Adler & Davis, 2006) because researchers (e.g., Ambrose, 2004; Hiebert, 1999; Hill & Ball, 2004; Lloyd & Wilson, 1998; Rittle-Johnson & Kroedinger, 2002) have found that procedures are formed and developed from students' sometimes faulty prior conceptual understanding of mathematical concepts. The process of integrating procedural knowledge with conceptual knowledge is difficult for many pre-service teachers who were taught in a traditional method and learned mathematics using rote memorization (Hill & Ball, 2004). To ensure teachers are equipped to teach both procedurally and conceptually, teachers themselves need to understand how students learn, comprehend multiple solutions and methods, and provide models to represent various mathematical concepts (Hill & Ball, 2004). Ensuring conceptual and procedural knowledge in students requires that teachers themselves have a rich and well-connected conceptual understanding of mathematics (Hill & Ball, 2004).

Teacher Preparation Programs

In the 1800s, when the idea of public schools became prevalent in the United States, many of the teachers knew only slightly more that the children they were expected to teach (Wise & Leibbrand, 2000). This lack of knowledge was partly due to the fact that, in the 1800s, the workforce did not require workers who were highly educated, but those who possessed basic skills such as reading and writing (Wise & Leibbrand, 2000). By the early 1950s, teaching preparation and requirements were lacking substance and many thought of the teaching profession as a job that one could do, if they couldn't do anything else (Wise & Leibbrand, 2000). In 1983, the report A Nation at Risk was published, which was considered the foundation for improvement in the U.S. education system (Wise & Leibbrand, 2000). By 1987, the National Council for Accreditation of Teacher Education (NCATE) created a framework that outlined the specific courses and knowledge that teachers needed to be successful (Wise & Leibbrand, 2000). This framework sparked the need for clinical practice and performance assessments to determine whether educators had the adequate knowledge needed for teaching (Wise & Leibbrand, 2000). Despite this push for an increase in teacher quality, policy makers were still concerned with American students' performance on international tests (Wise & Leibbrand, 2000). U.S. student's poor academic achievement has now become the focus of teacher quality and teacher preparation programs (Wise & Leibbrand, 2000).

Teacher Preparation and Math Achievement

Researchers have found that a teacher's knowledge of mathematics could help to determine students' likeliness to understand mathematics (Kajander, 2010). Many researchers, such as Kajander (2010) determined that many elementary teachers had a weak understanding of the concepts they were expected to teach. Kajander (2010) believed that these weaknesses in teacher knowledge is are direct reflection of their teacher preparation courses (Kajander, 2010). Kajander (2010) believed that teachers needed to be taught how to teach mathematics differently than how they were taught when they were in school. Thus, teachers need to deepen their conceptual understanding of mathematics far beyond how they were taught (Kajander, 2010). However, Boerst et al., (2008) found success in students even when they were taught by teachers with little to no teacher preparation (Boerst et al., 2008). This study found that actually teaching the students, regardless of the teacher's credentials, was more important than going through a teacher preparation program (Boerst et al., 2008). This finding contradicts what many researchers believe about the importance of teacher preparation programs. Educational researchers such as Benner and Hatch (2009) argued that in order to increase mathematics achievement, teachers must be properly prepared. Additionally, Conklin (2007) stated that teacher preparation programs did not properly prepare pre-service teachers for the challenges of effectively teaching children. Conklin (2007) found that although many teacher preparation programs teach pre-service teachers methods for teaching effectively, they are not adequately shown how to use these methods in actual classrooms. Therefore, many pre-service teachers are not fully prepared to teach upon completion of the teacher preparation program (Conklin, 2007).

Chapter 3: Research Methods

Purpose

The purpose of this study is to investigate the various types and frequency of errors made by elementary pre-service teachers in mathematics. Additionally, this study examined whether relationships exist between error type and cognitive domain or between error type and content domain.

Research Questions

This study investigated the following research questions:

1) What types of errors are made by elementary pre-service teachers in mathematics?
2) What is the frequency of these errors?
3) Is there a relationship between types of errors and cognitive domain?
4) Is there a relationship between types of error and content domain?

Research Design

This error analysis utilized free-response questions adapted from the TIMSS 2011 assessment at the fourth grade level. Relationships between types of questions, types of errors, and cognitive domain were investigated using a chi-square analysis. Participation in this study was voluntary in that professors were asked if their elementary pre-service mathematics or mathematics methods classes would participate in the study. Students in those classes were asked to participate voluntarily with no compensation. Participants were given a revised version of the TIMSS assessment (Appendix B). The revised assessment consisted of free-response questions and multiple-choice questions that were rewritten into free-response form on the TIMSS assessment. These TIMSS assessment items were categorized by topic and by cognitive domain. The assessment includes only number sense questions at the elementary level. Number sense questions were chosen because number sense is the gateway to algebra and thus, an in-depth understanding of number sense is necessary for successful understanding of algebra (Mullis et al., 2011). The specific types of questions assessed in this study included

1. Fraction and decimal
2. Whole number
3. Number sentences with whole numbers
4. Patterns and relationships

These specific types of questions were chosen because these number sense topics were presented in the original TIMSS assessment. These questions were assessed on all three cognitive domain levels presented in the original TIMSS assessment:

1. Knowing
2. Applying
3. Reasoning

This assessment provided the quantitative means to investigate the types of errors committed by each participant, the relationship between error type and cognitive domain, and the relationship between error type and type of question.

Participants

Participants in the study were fifty-five pre-service elementary education undergraduate students at a public, coeducational university located in Georgia. All participants were 18 years or older and were currently enrolled in the mathematics methods class (ECE 4401) or the final mathematics classes for elementary education (MATH 3318). The professors of all of the sections of those courses were invited to participate. Of the eight professors asked, only five agreed. Most of the other methods instructors explained that their curriculum was too full for them to give up 45 minutes of class time. The students in the classes of the professors who agreed to participate were given the choice to participate in the study and then completed a consent form indicating their willingness to participate in the study. All students in each participating class agreed to participate in the study.

Methods

The assessment was administered during the first half of the 2015 spring semester by a mathematics education faculty member. The purpose of the study was explained to each participating class. Additionally, the assessment itself was explained to the participants, outlining the types of questions and where the questions came from. Students were then asked to complete the consent form, and after all consent forms were completed, the professor administered the assessment (appendix A). All participants were given 35 minutes to complete the assessment.

The assessment contained 32 number elementary mathematics questions (Appendix B). All questions were in free-response form. The questions used were chosen based on the percentage of correct responses received from U.S. students. All "number" questions indicating that less than 100% of U.S. 4th graders understood the concept were included in the teacher assessment (teacher assessment shown in appendix B). Additionally, all questions conducive to free-responses were rewritten in free-response form thus allowing for an error analysis to be conducted. Any multiple-choice question that could not be easily written in free-response form was discarded because the error analysis required teacher candidates to show their solution process.

Original TIMSS Question:

A train left Redville at 8:45 a.m. It arrived in Bedford 2 hours and 18 minutes later. What time did it arrive in Bedford?

A. 11:15 a.m.
B. 11:13 a.m.
C. 11:03 a.m.
D. 10:53 a.m.

Revised TIMSS Question:

A train left Redville at 8:45 a.m. It arrived in Bedford 2 hours and 18 minutes later. What time did it arrive in Bedford? Explain your answer.

Original TIMSS Question:

Which of these fractions is larger than $\frac{1}{2}$?

A. $\frac{3}{5}$
B. $\frac{3}{6}$
C. $\frac{3}{8}$
D. $\frac{3}{10}$

***A question like this was not included in the revised assessment because the test taker would need to see the choice to obtain the correct answer.

Although some of the questions were rewritten, the integrity of the questions remained intact. Participants were given 35 minutes to complete the assessment (without a calculator) in which they were required to show all work even when the question was perceived as easy. Two versions of the test were created in which the order of the test questions were shuffled. This was done so that if the test takers ran out of time on the assessment, the same question would not be missed repeatedly. Number questions were chosen for the test because the bulk of the questions provided in the TIMSS released assessment focused on number (Mullis et al., 2011). Additionally, number is a key prerequisite for successful understanding of algebra in later years (Mullis et al., 2011).

Error Analysis

Fleishchner and Manheimer (1997) see error analysis as a tool used to analyze mathematical work to determine areas of weakness. Error analysis is used primarily to examine mathematical mistakes made by students (Fleishchner & Manheimer, 1997). Error analysis is far more valuable than a numeric score because it categorizes the skills and processes that the student lacks as well as helps the assessor understand students' thinking, particularly misconceptions (Fleishchner & Manheimer, 1997).

Newman (1977) categorized five literacy and numeracy skills critical to understanding mathematics problems: reading, comprehension, transformation, process skills and encoding. Newman's Error Analysis (NEA) laid the groundwork for understanding why students experience difficulties when solving mathematical problems. Additionally, NEA provided a way for teachers to diagnose students' misunderstandings. However, NEA solely focused on error analysis in mathematical problem solving in word problems (Newman, 1977).

Error analysis in this study involves coding of student work based on the work of Meyers (1985) who adapted NEA to be applicable to more than just mathematical problem solving. This study used his six codes (Meyers, 1985) to categorize errors: number selection error, missing step error, operation error, computational error, random error, and omission error (Figure 8).

Upon reviewing each participant's test and coding the errors as described above, the categories of error types made on each problem were entered into a spreadsheet for further analysis. Using the sort functions of the spreadsheet software, the data was first sorted by question type. Template 1 in Appendix C illustrates how each question was broken down to classify each participant by their response. Similarly, Template 2 in Appendix C sorts the data

Figure 8

Classification and Examples of Errors

	Incorrect Answer
Number Selection Error using the wrong number or putting it in the wrong place	Georgia wants to send letters to 12 of her friends. Half of the letters will need 1 page each and the other half will need 2 pages each. How many pages will be needed altogether? $12 \times 2 = 24$ $\quad 12 + 24 = 36$
Missing Step Error completing fewer steps than needed to solve the problem	Steve had 15 sports cards to trade for animal cards. How many cards would he get? $15 \div 3 = 5$
Operation Error using the wrong operation to solve the problem	5,631+ 286 = Answer: 5325 $\quad$ 5,6311 + 286 = 5325
Computational Error calculation mistake	Tom ate ½ of a cake, and Jane ate ¼ of the cake. How much of the cake did they eat altogether? $\frac{2}{6}$ $\quad 4 + 2 = 6$ $\quad \frac{1}{2}\ \frac{1}{4} = \frac{2}{6}$
Random Error wrong without justification	Write a number that is larger than 5 and is smaller than 6? 6
Omission Error the question is left blank	Tom ate ½ of a cake, and Jane ate ¼ of the cake. How much of the cake did they eat altogether? Justify your answer. ?

by participant to show the types of errors the participant demonstrated for each question. Lastly, template 3 listed the error types by question to determine the overall percent of each error occurring for each question. Templates 1, 2 and 3 (Appendix C) addressed the first two research questions in that they showed the types of errors as well as the frequency of those errors for each participant and for each question.

Following an analysis of the types of errors and the frequency of those errors, templates 4 and 5 were used to investigate whether a relationship exists between cognitive domain and error type as well as question topic and error type. This analysis will answer the third and fourth research questions in that it organized the data to investigate any possible relationships between cognitive domain, error type and question topic.

In all, the data gathered from the participants were analyzed qualitatively. The researcher, and two other analysts (one of whom is a mathematics education doctoral student, and the other who has an extensive knowledge of mathematics) read through all of the participants' responses. The responses were then analyzed and coded by error type. The researcher then tabulated and categorized the error codes indicated from the analysis. The use of the additional two analysts helped to ensure that the coding was reliable. This form of interrater reliability is known as triangulation through multiple analysis (Patton, 1990). Through analyst triangulation (using multiple analysts), the risk of bias by the researcher is greatly reduced therefore increasing the validity and reliability of the data analysis (Patton, 1990). In analyzing interrater reliability, a Cohen Kappa of 0.87813 was obtained. This result indicates that the coding of errors were reliable (Landis & Koch, 1977).

Statistical Method of Analysis: Chi-Square

A chi-square test is used to determine if whether a relationship exists between categorical variables. Using a two-way table, observed counts are compared to expected counts of each cell in the table. From there, a chi-square test statistic (p-value) will be used to measure the deviation between observed counts and expected counts for each cell of the table. Thus, the chi-square test will be used to test the following hypotheses:

H_0: There is no association between error type and cognitive domain

Ha: These is an association between error type and cognitive domain

and

H_0: There is no association between error type and content domain

H_a: These is an association between error type and content domain

Specifically, this test will determine whether the difference between expected counts and observed counts is large enough to be statistically significant. In other words, this test will show whether the interactions between the two variables are what we would expect given the number of errors identified. A χ^2 test statistic and a p-value will be computed to provide evidence against the null hypothesis and in favor of the alternative hypothesis. A p-value less than $\alpha = 0.05$ will provide enough evidence to reject the null hypothesis and conclude the alternative hypothesis. Similarly, a p-value is greater than $\alpha = 0.05$ will provide enough evidence to not reject the null hypothesis and to conclude insufficient evidence for the alternative hypothesis.

The chi-square distribution is an approximation to the normal estimation used in a binomial distribution. As cell counts increase, the approximation becomes more accurate. Therefore the accuracy of this calculation is determined by the size of the expected counts for each cell. Specifically, at least 80% of the expected cell counts must be greater than five. If this condition does not hold true, rows or columns can be combined or deleted to ensure that the results of the test are valid.

Limitations and Delimitations

A limitation of this assessment was that many of the questions on the TIMSS assessment were originally written in multiple-choice format. Thus, changing the format to free-response form arguably changed the difficulty of the question by removing the guessing option from the problem. Therefore, error rates of student and pre-service teachers cannot be compared outright.

The findings of this research are also limited. Many students did not complete the entire assessment and therefore it was hard to determine which items they did not attempt because of time and which they did not attempt do because they did not know how. Therefore to investigate the errors using statistical measures, only the students' incorrect responses were recoded. Missing responses that were left blank were excluded from the statistical analysis because students' errors cannot be identified from a blank response (Wijaya, 2014). Furthermore, the expected cells count condition of the chi-square test was not initially satisfied. There were too many expected cell counts less than five and therefore the researcher decided to delete the two least common error type categories to satisfy this condition (random error and number selection error).

Ethical Consideration

This study was approved by the university's Institutional Review Board (IRB). Thus, the researcher in this study obtained informed consent, ensured privacy and confidentiality, and maintained the integrity of the data. The assessment results of each participant in this study remained confidential. The following procedures were used to protect the confidentiality of the study records: The study records were kept in a password-protected electronic file, the records were labeled with a code, no names were used in the study, and any files containing identifiable information were safely secured. Additionally, students were informed (via the informed consent) that at the conclusion of this study, the findings may be published.

Chapter 4: Findings

Introduction

Pre-service teachers were given 35 minutes to complete the revised TIMSS assessment. Some students completed the assessment early, while some students worked for the entire 35 minutes. Pre-service teachers' results on the assessment varied within each class. The data showed that some students in each class performed very well on the assessment, while others struggled. Despite the various levels of understanding presented in each class of pre-service teachers, some trends were apparent among all of those participating in this study and some common strengths and weaknesses were demonstrated. In all, the data showed that many of these teachers struggle with the mathematical concepts in the 4th grade curriculum. Additionally, the majority (60%) of the pre-service teachers left questions on the assessment blank (on average six questions), which further indicates that these teachers may have struggled with the topics presented on the assessment or with completing them within the time that would have been given to fourth grade students taking the TIMSS assessment.

In all, 44% of the pre-service teachers taking this assessment answered 80% or more of the questions correctly. Additionally, 18% of these pre-service teachers scored 90% or higher. While every pre-service teacher answered at least six questions correctly, only two pre-service teachers earned a perfect score on the assessment. In contrast, six pre-service teachers of the total 55 (11%) answered less than 1/3 of the questions correctly on this assessment. Additionally, the student earning the lowest score of a 22% only answered seven questions correctly and omitted all of the rest but three. The average percentage of correct responses given on this assessment by the pre-service teachers was 67.2% However, this data was skewed left and therefore the median would be a better indicator of the typical score earned by a pre-service teacher on this assessment. The median score was 71.9%. The middle 50 % of the data ranged from a score of a 50% to a score of 87.5%. The standard deviation for this data set was 21.8% which indicates that on average the scores deviate from the mean by 21.8%.

Figure 9
Histogram of the Percentage Correct of the Pre-service Teachers

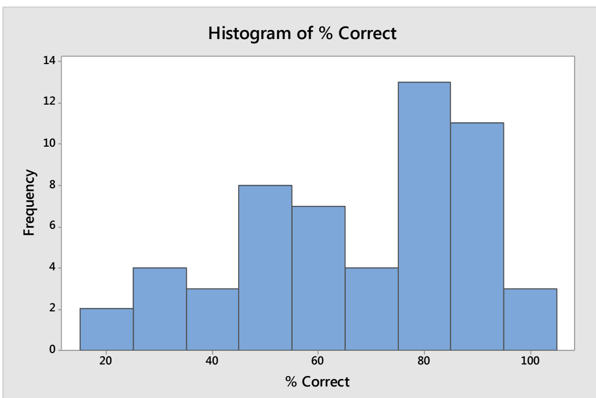


Figure 10
Boxplot of the Percentage Correct of the Pre-service Teachers

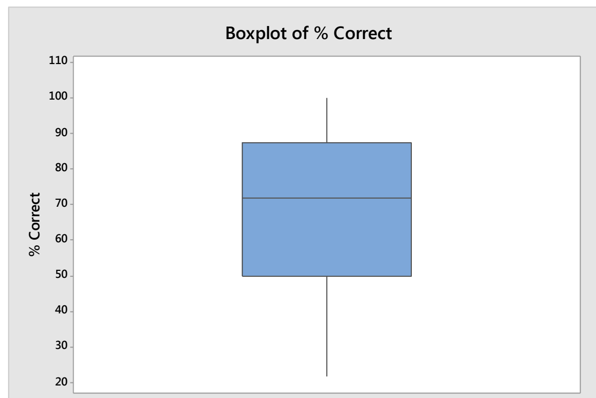


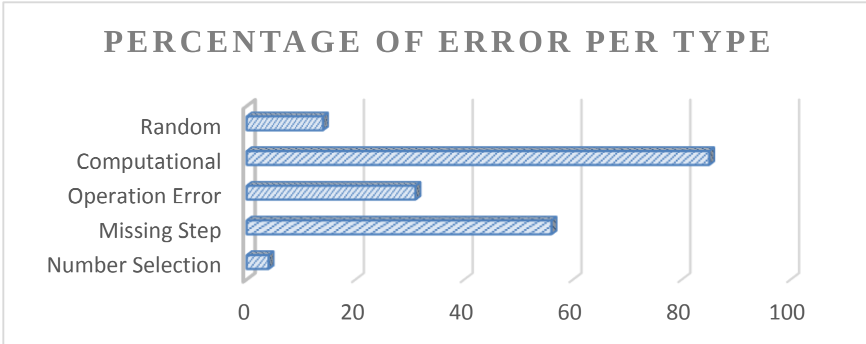
Table 1
Descriptive Statistics of the Percentage Correct of the Pre-service Teachers

Variable	Mean	StDev	Minimum	Q1	Median	Q3	Maximum	IQR
% Correct	67.27	21.78	21.88	50.00	71.88	87.50	100.00	37.50

Error Analysis

Errors on the adapted TIMSS assessment were classified into five categories in the error analysis phase: number selection error (N), missing step error (MS), operation error (OP), computational error (C), and random error (R). Student results were scored, classified and then each question was sorted by error.

Figure 11
Percentage of Error per Type



In all the students made 190 errors on the assessment with an average of three errors per student (Table 2). Table 2 shows the percentage of each type of error made by the students in this study. The analysis of responses reveals that of the 190 incorrect responses, 2% were number selection errors, 7% were random errors, 16% were operation errors, 29% were missing step errors, and 45% were computational errors, Additionally, 62% of the questions were omitted. Of the students who committed errors after attempting the question, most of these students made computational or missing step errors in their attempt (Table 2). In the following section, each of these types of errors will be described and an example will be discussed.

Table 2
Frequency of Error Types

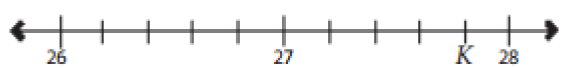
Type of Error	N	%
Number Selection Error	4	2
Missing Step Error	56	29
Operation Error	31	16
Computational Error	85	45
Random Error	14	7
Total of Observed Errors	190	100

Observed Number Selection Error

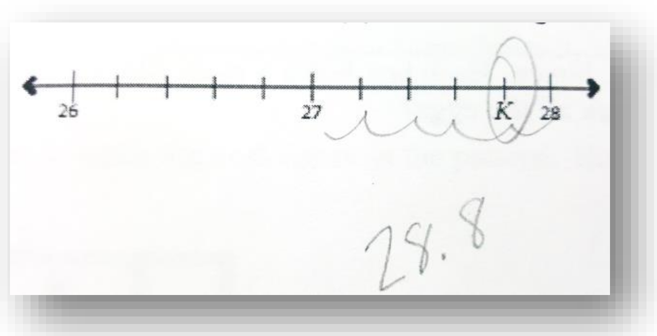
Number selection errors were rarely made by the pre-service teachers on this assessment. Number selection errors only made up one percent of the total errors committed in this study. This result indicates that students chose the correct numbers to use to solve the problems in the assessment. Although relatively uncommon, number selection errors appeared more often in number sentences with whole number question types and at the *applying* cognitive domain level. Question 32 had the highest amount of number selection errors than any other question on the test. However this question was a knowledge-level question regarding fractions and decimals. Example 2 shows an example of student work which contains a number selection error (Figure 12). The student had to determine what number K represented on the number line, which assessed their understanding of fractions and decimals.

Figure 12
Example of a Number Selection Error

Question 32 What number does K represent on this number line? Justify your reasoning.



Sample Student Response



The student seemed to accurately count the decimal markings on the number line, however, they chose the incorrect whole number. Instead of recording their answer as 27.8, they recorded their answer as 28.8, thus selecting the wrong whole number to form their answer. It is possible that this student did not understand number lines, however with the minimal work shown, it is difficult to tell.

Observed Random Error

Similar to number selection error, random error was rarely demonstrated by the pre-service teachers taking this assessment. Either the teacher candidates answered the questions correctly or the errors that they committed were obvious and their work could be followed to determine the type of error. Of the total errors committed by pre-service teachers in this study, only 3% of the errors were classified as random errors. These errors could not be classified in any other category because either work was not shown or was too hard to follow.

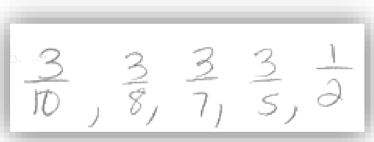
Random errors were distributed fairly evenly across the cognitive domain levels (four at the knowledge level, five each at the applying and reasoning levels). However, random errors occurred more frequently in whole number problems than in any other type of problem (seven random errors in whole number problems). Question 18, which was a fraction and decimal

Figure 13
Example of a Random Error

Question 18 – Write these fractions in order from least to greatest. Explain your reasoning.

$$\frac{1}{2}, \frac{3}{5}, \frac{3}{10}, \frac{3}{8}, \frac{3}{7}$$

Sample Student Response:



reasoning problem, was the source of most of the random errors found on the assessment (four errors). Students answering this question often provided no rationale for their reasoning. Many students provided an answer with little to no explanation, therefore yielding a random error (Figure 13). Although the coders hypothesized about the errors made on this problem, student work provided no evidence and so the error was categorized as random.

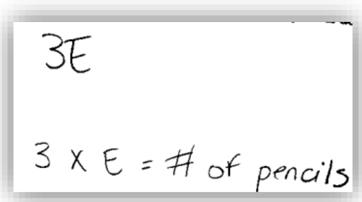
Observed Operation Error

Pre-service teachers more often made operation errors on this assessment, typically when students used the wrong operation to solve the problem. Most of the operation errors came from whole number (including number sentences with whole numbers) application problems. Question 13 showed the highest number of operation errors on the assessment (Figure 14). This question involved number sentences and application of whole numbers.

Figure 14
Example of an Operation Error

Question 13 - E stands for the number of pencils Pete had. Kim gave Pete 3 more pencils. How many pencils does Pete have now?

Sample Student Response



Specifically 1/3 of the students who answered this question incorrectly committed an operation error (10 students out of 31 students).

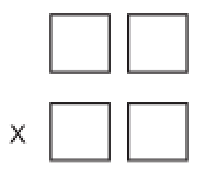
Observed Computation Error

Computation errors were found fairly often throughout each assessment. Students often used the correct operation but then made mathematical mistakes therefore producing computational errors. Computational errors occurred most often in whole number application problems. Basic mathematics mistakes seemed to plague some students when attempting to compute application problems involving whole numbers. Question 29 had the highest number of computational mistakes (Figure 15). Additionally question 18 exhibited several computational errors with students (Figure 16).

Figure 15

Example of a Computational Error

Question 29 - Place the four digits 3, 5, 7, and 9 into the boxes below in the positions that would give the greatest result when the two numbers are multiplied.



Sample Student Response:

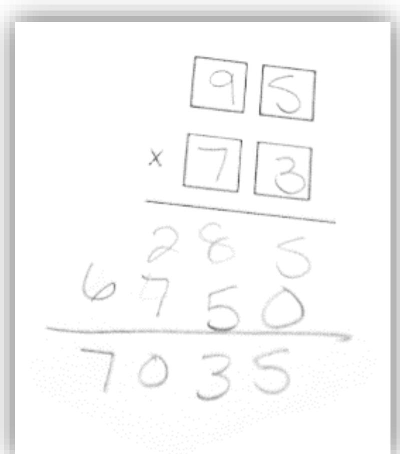


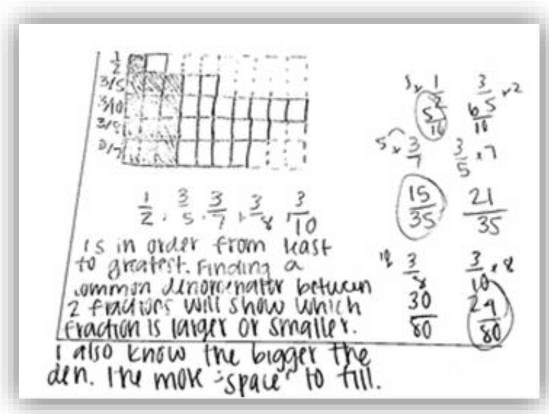
Figure 16

Example of a Computational Error

Question 18 - Write these fractions in order from least to greatest. Explain your reasoning.

$$\frac{1}{2}, \frac{3}{5}, \frac{3}{10}, \frac{3}{8}, \frac{3}{7}$$

Sample Student Response:



Although these questions are not of the same type or cognitive domain, both of these questions involved the ordering of numbers. It is clear that these pre-service teaches made computational errors when it came to putting numbers in the correct order relative to size, whether it be fractions and decimals or whole numbers.

Observed Missing Step Error

The participating pre-service teachers made missing step errors quite frequently. This type of error mostly occurred when the student did not answer the question fully. For the most part, students answered part or most of the question, but failed to answer the question completely. Most students committed a missing step error in whole number knowledge questions. Question 15, a question on whole numbers at the knowledge level, had the most missing step errors made by the pre-service teachers taking this exam (Figure 17).

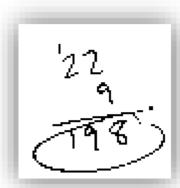
Figure 17

Example of a Missing Step Error

Question 15 – Multiply the following expressions. Round your answer to the nearest hundred.

$$9\ x\ 22$$

Sample Student Response



Students failed to completely answer the question by failing to round the answer and therefore made a missing-step error.

Summary

In all, the pre-service teachers who participated in this study were most likely to commit computational errors or missing step errors. On the other hand, this may be misleading because of the number of omitted problems on the assessment. On average, teachers omitted 6 questions out of 32 on their assessment.

Statistical Analysis: Chi-Square Test for Association

The chi-square test for association is used to test various categorical variables. This test determines whether there is a relationship between categorical variables in a two-way table. The alpha level for this study was set at 0.05. P values less than $\alpha = 0.05$ will provide enough evidence to reject the null hypothesis and conclude the alternative hypothesis, that there is enough evidence to reject the null hypothesis and conclude that there is an association between the variables. However, before running the test, the 80% condition for expected cell counts must be satisfied. In other words, 80% of the cells in the table must have expected counts larger than five.

Five error types (omission errors were excluded from the analysis) were analyzed in this section to check for relationships with level of cognitive domain and the question type (Table 3). In an initial test, it was noted that more than 20% of the cells had expected counts less than five.

Table 3

Cognitive Domain by Error Type

Error Type	**Cognitive Domain**		
	Knowing	Applying	Reasoning
Number Selection	1	3	0
Missing Step	23	13	20
Operational	6	13	12
Computational	14	49	22
Random	4	5	5

Therefore number selection errors and random errors were removed from the analysis, since very few errors were shown in those categories. Tables 5 and 6 show the Minitab output for these test with the combination of these two variables.

Table 5

Results of Chi-Square Test for Association: Error Type by Cognitive Domain

Error Type	**Cognitive Domain**		
	Knowing	Applying	Reasoning
Missing Step	23	13	20
	(3.382)	(−3.747)	(0.848)
Operational	6	13	12
	(−0.802)	(−0.207)	(0.969)
Computational	14	49	22
	(−2.553)	(3.671)	(−1.540)

Note. $\chi^2 = 19.541, df = 4$. Numbers in parenthesis indicate adjusted standardized residuals. $* p = 0.001$

The first chi-square test attempted to answer the third research question: Is there a relationship between types of errors and cognitive domain? Therefore the null and alternative hypotheses for this test were:

H_0: There is no association between type of error and cognitive domain

H_a: These is an association between type of error and cognitive domain

The test of association results indicate that the type of error the pre-service teachers committed does appear to be statistically associated with the level of questions presented on the TIMSS assessment ($\chi^2(4, N = 172) = 19.541; p = 0.001$). The results show a statistically significant difference in error types between knowing, applying and reasoning problems. These results suggest that the type of error committed is different for each level of question. Specifically, in the knowledge cognitive domain there were more missing step errors than expected. Additionally, in the applying cognitive domain, there were more computational errors than expected. Lastly, in the reasoning cognitive domain, there were more operational errors than expected.

Similarly, the second chi-square test was run to answer the fourth research question: Is there a relationship between types of error and question type (Table 4)? Therefore, the null and alternative hypothesis for this test was:

H_0: There is no relationship between type of error and question type.

H_a: These is a relationship between type of error and question type.

Table 4
Content Domain by Error Type

	Content Domains			
Error Type	Whole Numbers	Patterns and Relationships	Number Sentences with Whole Numbers	Fractions and Decimals
Number Selection	1	0	3	0
Missing Step	29	3	16	8
Operational	16	6	7	2
Computational	30	14	32	9
Random	7	3	4	0

The test of association results indicate that the type of error the pre-service teacher committed does not appear to be statistically associated with the type of questions presented on the TIMSS assessment ($\chi^2(6, N = 172) = 9.846; p = 0.131$). The results show no statistically significant difference in error type between whole number, patterns and relationship, number sentence, and fraction and decimal problems (Table 6). These results suggest that the type of error committed is similar for each question type.

Table 6

Results of Chi-Square Test for Association: Error Type by Content Domain

	Content Domain			
Error Type	Whole Numbers	Patterns and Relationships	Number Sentences	Fraction and Decimals
Missing Step	29 (1.503)	3 (−2.146)	16 (−0.665)	8 (0.942)
Operational	16 (0.993)	6 (1.081)	7 (−1.239)	2 (−0.901)
Computational	30 (−2.172)	14 (1.180)	32 (1.576)	9 (−0.190)

Note. $\chi^2 = 9.846, df = 6$. Numbers in parenthesis indicate adjusted standardized residuals.
$* p = 0.131$

Summary

In all, the chi-square analysis shows an association between types of error and cognitive domain but not between types of error and types of question. This result indicates that the participants in this study made similar types of errors on similar levels of problem but not on similar question types. Furthermore, the relationship shown between cognitive domain and error type is statistically significant and therefore could not have happened by chance.

Chapter 5: Summary, Discussions and Conclusion

Introduction

The purpose of this research was to explore the errors made by pre-service elementary teachers on an international assessment given to elementary school students. The content focus of the assessment was number. This topic was chosen as a focus because it is a pre-requisite skill needed for success in algebra, a topic that students generally struggle with based on research (Pappano, 2012). In conducting this research, my goal was to develop a richer understanding of the readiness of pre-service elementary teachers to teach mathematics, specifically the types of errors that they make, on the elementary mathematics topic of number. In the next sections, I will revisit the research questions presented in this dissertation, discuss the errors made by teachers in this study, and reflect on my theoretical framework. Finally, I will reflect on the study and examine further possibilities for future research.

Discussion of Findings

In this dissertation, I explored the following four research questions:

1) What types of errors are made by elementary pre-service teachers in mathematics?
2) What is the frequency of these errors?
3) Is there a relationship between types of errors and cognitive domain?
4) Is there a relationship between types of error and question type?

An error analysis conducted in this study was used to answer these questions. Additionally, a chi-square test for association was used to determine if there was a relationship between error type and cognitive domain and error type and question type. Throughout the chapter, examples of student errors are shown to demonstrate the types of errors pre-service teachers made on this revised adaption of TIMSS assessment items.

Teacher knowledge Examined Through Error Analysis

The errors exhibited by the pre-service teachers in this study included all of those in Meyers' (1985) categorization: number selection errors, missing-step errors, computation errors, random errors, operation errors, and omission errors. It is clear that these pre-service teachers struggle most with missing step errors (30%) and computational errors (45%). The number of these errors calls into question these pre-service teachers' knowledge of number (Hill et al, 2008).

Understanding the reasons or causes of each error during the error analysis phase is crucial (Peg & Luo, 2009). Some errors found in the error analysis phase of this research indicated that pre-service teachers knew the content, but either made a basic computational mistake or did not answer the question fully. Riccomini (2005) classified these errors as errors that only happen once, versus habitual errors. These errors were generally classified as number selection errors or missing step errors. These pre-service teachers demonstrated that they knew how to solve the problem but made a non-conceptual mistake somewhere in their work. Similar to Brodie's (2005) findings some of these errors were expected and understandable. Therefore, although these errors were somewhat alarming, they did not indicate that these pre-service teachers have a lack of conceptual understanding of basic number sense.

Figure 18 shows an error committed by a pre-service teacher in this study. Although this error was classified as a number selection error because the teacher candidate chose to use 11:45 instead of 10:45, this error seemed to be a simple mistake of using the wrong number in their calculation. Since all of the pre-service teacher's work shown was correct, and he or she simply chose the wrong number to use in their final calculation, this error would not be considered a lack in conceptual understanding.

Figure 18

Example of a Non-Conceptual Error

A train left Redville at 8:45 a.m. It arrived in Bedford 2 hours and 18 minutes later. What time did it arrive in Bedford?

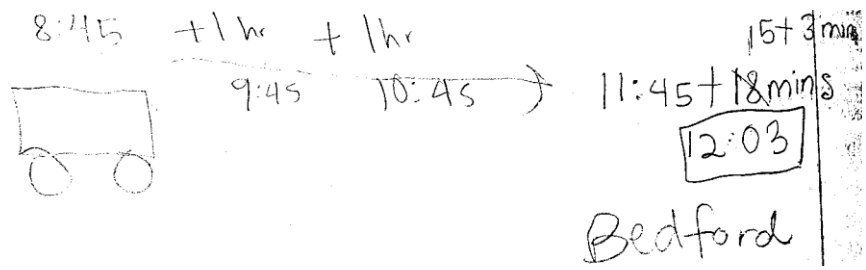


Figure 19 shows another instance where a pre-service teacher made a simple mistake, but the mistake did not indicate a lack of conceptual understanding. In this example, the pre-service teacher was asked to determine the smallest three-digit number that could be made with the numbers given. The rationale provided by the pre-service teacher was correct, that in order

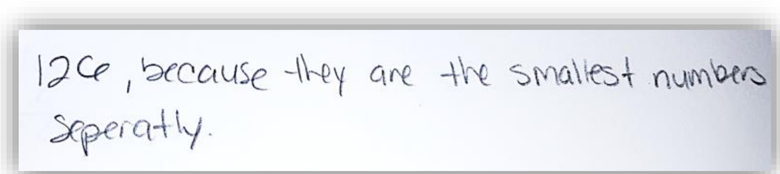
Figure 19

Example of a Non-Conceptual Student Error

Anna has these cards with numbers on them.

1	8	6	5	2

What is the smallest three-digit number she can show with the cards? She may use each card only once. Justify your answer.



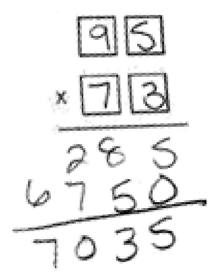
to create the smallest three-digit number, the three smallest numbers would need to be used. However, it seems as though the pre-service teacher simply overlooked one of the smallest numbers and therefore chose the next smallest number, which yielded him or her an incorrect answer. This error was classified as a number selection error, however it did not indicate a lack of conceptual understanding by the pre-service teacher.

On the other hand, many of the computational errors found in this error analysis showed that the pre-service teachers who committed these errors lacked conceptual understanding of elementary mathematics. These errors were critical, as research shows that teachers need a deep understanding of the mathematics they are expected to teach (Gadanidis & Namukasa, 2007). These errors can help to determine areas in which these pre-service teachers may not have the necessary mathematical knowledge to be effective in the classroom (Peng & Luo, 2009).

Figure 20 illustrates an instance where the pre-service teacher demonstrated a lack of conceptual knowledge regarding rounding as well as a lack of mathematical reasoning. Although this pre-service teacher committed a computational error, he or she also showed, by the numbers chosen, that a lack of understanding of how to obtain the largest number. The mistake demonstrated by this one pre-service teacher was demonstrated by others. In fact, this question was the most missed question on the assessment. Of the pre-service teachers, 78% either didn't answer the question at all, or answered it incorrectly. Out of the 43 pre-service teachers who did not earn credit for this question, 63% chose the numbers 95 and 73 as the two numbers that would yield the largest product. Riccomini (2005) would have classified this type of error as habitual amongst the pre-service teachers. The consistency among the pre-service teachers in choosing the wrong combination of numbers seems to show a misconception that the largest single number would yield the largest overall product. One solution would involve rounding the numbers 95 and 73 to gives 100 x 70. This product is 7000. On the other hand, if the pre-service teachers chose the number 93 and 75 and rounded appropriately, they would have obtained 90 x 80 which yields a product of 7200. This product is clearly larger and thus the correct answer to this problem is 93 x 75. This problem shows that these pre-service teachers may lack conceptual understanding of the nature of multiplication or of the distributive property, as well the use of rounding.

Figure 20
Example of a Conceptual Student Error

Place the four digits 3, 5, 7, and 9 into the boxes below in the positions that would give the greatest result when the two numbers are multiplied.



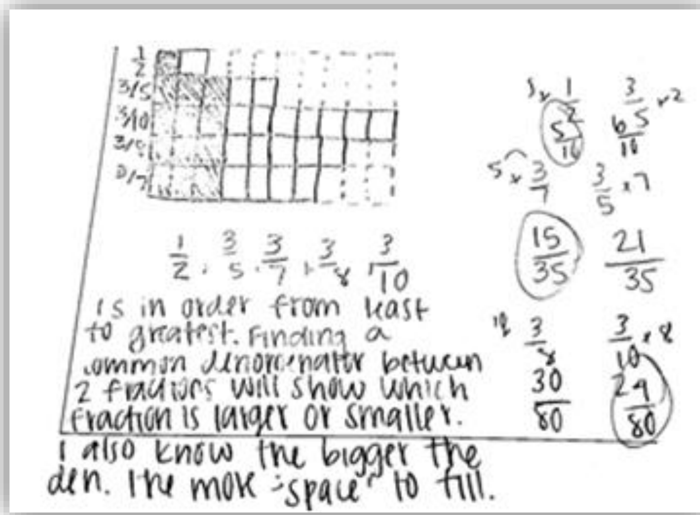
This next example shows how conceptual knowledge requires a deep understanding of mathematical relationships and a structural understanding of mathematical ideas (Eisenhart, Borko, Underhill, Brown, Jones, & Agard, 1993). Figure 21 shows how this pre-service teacher had a limited understanding of fractions. In this pre-service teacher's explanation, he or she discussed the need for a common denominator. Although finding a common denominator in this problem could yield a correct answer, trying to find a common denominator for 2, 5, 7, 8, and 10 could be somewhat time consuming and so is not a very efficient strategy.

Figure 21

Example of a Conceptual Student Error

Write these fractions in order from least to greatest. Explain your reasoning.

$$\frac{1}{2}, \frac{3}{5}, \frac{3}{10}, \frac{3}{8}, \frac{3}{7}$$



Although this method could work, the difficulty would be in finding a common denominator for all five fractions. This pre-service teacher simply found a common denominator between pairs of fractions, resulting in an incorrect answer. This pre-service teacher showed a limited understanding of common denominators, and no understanding of the value of common numerators. Had the pre-service teacher changed the $\frac{1}{2}$ to $\frac{3}{6}$, he or she could have ordered the fractions using the denominators. Again, this demonstrates limited mathematical knowledge that many researchers such as Hill (2008), Ball (2008), and Shulman (1986) deemed necessary for effective teaching.

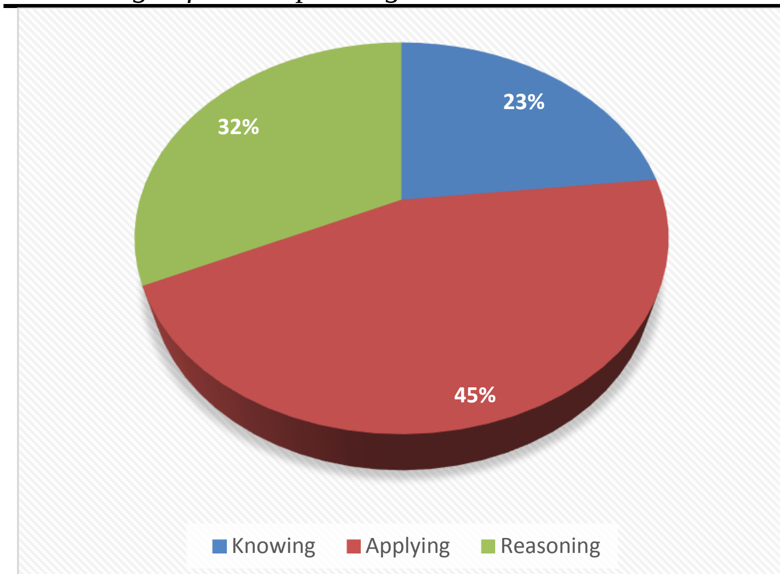
Some of the errors committed in this study that show a lack of conceptual understanding, indicate that the content knowledge of these pre-service teachers needs to be addressed. With content knowledge being the foundation for PCK and MKT (e.g., Hill et al, 2008), these limitations need to be addressed before they can become effective mathematics teachers.

Errors within Each Cognitive Domain

A chi-square test for association was used to determine whether an association existed between error type and cognitive domain. Unlike Toker's (2010) study, this study found a statistically significant association between types of errors and cognitive domain (Figure 22). This test indicated that in the knowledge cognitive domain, there were more missing step errors than expected. Knowing questions involve recall of facts, concepts and procedures (Mullis, 2003), therefore it is not expected that missing step errors in this domain would be that high. Similarly, this test indicated that there were more computational errors in the applying cognitive domain than expected. The applying cognitive domain requires application of knowledge and conceptual understanding (Mullis, 2003). The number of conceptual errors shown in this domain indicates limitations to pre-service teachers' conceptual understanding of the mathematics concepts presented on this revised TIMSS assessment. Reasoning, the third and highest domain of the problems on in the TIMSS assessment, required that the student go "beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multi-step problems" (Mullis, 2003, p. 140). Missing step and operational errors were higher than expected in the reasoning domain. This result indicates that these pre-teachers struggle with choosing the correct operation and completing the problems when the questions get harder.

Similar to the study conducted by Tatsuoka (2004), this study found that these teachers showed more weakness in some content areas than in others. Specifically, these pre-service teachers demonstrated more weakness in the applying cognitive domain than in any other domain. However, in contrast to Chen, Gorin, and Thompson (2008) these pre-service teachers were not strong in any particular cognitive domain. This study accurately aligns to the study conducted by Zonts (2013) in that both studies indicates a clear weakness in the applying and reasoning cognitive domains.

Figure 22
Percentage of Errors per Cognitive Domain



Additionally, a chi-square test for association was conducted on error type and content domain. This test found no statistically significant association when relating error type and content domain. This test failed to show that specific errors are associated with specific question types.

Teacher Preparation Programs

The purpose of this study was to examine pre-service teacher understanding of number in elementary mathematics. The error analysis used in this study proved to be appropriate in determining the types and frequency of errors (Peng & Luo, 2009). Furthermore, the chi-square analysis helped to show a relationship between the categorical variables used in this study. The results of this study raise questions about the mathematical knowledge of these pre-service teachers who participated in this study. Of the 55 pre-service teachers who took the assessment, 44% scored 80% or higher and, of those, 18% scored 90% or higher. Only two student earned a perfect score on the assessment. In contrast, six pre-service teachers of the 55 correctly answered less than 1/3 of the questions correct. The pre-service teacher earning the lowest score answered only 7 (22%) questions correctly, omitted 22, and committed 3 computational errors.

It seems, if the United States wants to improve mathematical understanding of our elementary students, and consequently, the scores of the students taking these international assessments (or any assessments for that matter), one approach is investing more time and effort into the foundational mathematics knowledge of the educators who are teaching these children (Ma, 1999). Research (Kajander, 2010) shows that the weak conceptual understanding of teachers is partly due to their teacher preparation program. Although most states require a test to assess pre-service teachers understanding of concepts, many times the test does not accurately assess the content knowledge of these teachers (NCTQ, 2009). In particular, some believe that the GACE (Georgia Assessment for Certification of Educators) does not provide a direct measure of pre-service teacher's content knowledge (NCTQ,2009). Specifically, with early childhood education majors, the GACE does not assess the teacher candidate's depth of knowledge nor does it assess the knowledge needed for effective teaching (NCTQ,2009). The content knowledge assessed on the early childhood education examine only weighs 30% of the entire test (GADOE, 2012). The systems currently in place seem insufficient to ensure that these teachers master the mathematical knowledge that they need in order to successfully teach mathematics (The National Commission on Teaching an America's Future, 1997). In addition to focusing on effective teaching strategies, pre-service teachers also need in-depth understanding of the curriculum and content that they have to teach and beyond (Hill, Sheep, Lewis, & Ball, 2007).

Conclusion

This study was based on Shulman (1986) as well as Hill, Sheep, Lewis and Ball (2007) in exploring the foundational mathematics knowledge of teachers underlying the PCK and MKT needed for effective teaching. Mathematics teachers need to have a sound foundation of the curriculum that they will teach before entering the classroom (Langham, Sundberg, & Goodman, 2006). This study was based on this theoretical framework of teacher knowledge and explored the readiness of pre-service teacher's knowledge to teach mathematics at the elementary level by determining their areas of weakness using an error analysis (Peng & Luo, 2009).

The findings of this study align closely to what other researchers have found in the past regarding the fundamental understanding of mathematics of pre-service elementary teachers. In particular, this study and the study conducted by Liping Ma (1999) shared similar results. Like Ma's (1999) research, this study attempted to explored teacher knowledge and its implications for U.S. students' lack of achievement compared to other countries in mathematics. Both this study and the study conducted by Ma (1999) found that limitations in the knowledge of school mathematics in U.S. elementary teachers.

Implications for Future Research

This study does not have to end with this work. Possible future directions include adapting these methods to assess the errors of students and their teachers. In assessing both students and their teachers, we could determine if the teacher's misconceptions and errors are influencing that of their students. Additionally, although students are taught by many teachers, we can examine the type of errors committed by the students on a particular concept and focus efforts on improving teaching strategies where the concept is first taught. To avoid the limitations in this study, any future work on this topic should be done using different assessments, designed for the purpose on improving content knowledge in elementary mathematics teachers, possibly Hill and Ball's (2005) assessment of mathematical knowledge for teaching. Although, the TIMSS assessment is a valid tool for assessing student knowledge, it was not designed for error analysis. Many of the questions on the assessment could be answered with little to no work and therefore, many of the pre-service teachers spent too much time trying to explain how they got the answer, possibly causing them not to finish all the items on the assessment.

Although this study provides useful, detailed results about the mathematical knowledge of these pre-service teachers, given the chance to conduct the study again, I would use different categories to code the errors. I found that understanding why the pre-service teacher committed particular errors is more important than the error itself. Thus, I would use an error categorization similar to Riccomini (2005) in that I would attempt to determine whether the error happened once or whether it was habitual. After determining if it was habitual, I would determine whether it was a conceptual error or procedural error (Elbrink, 2008). Furthermore, I would conduct interviews after the assessment to get feedback from the pre-service teachers to try to understand why they performed as they did (Newman, 1977). These interviews could potentially open up a number of other research areas regarding this topic, including pre-service teacher motivation, pre-service teacher beliefs, and pre-service teacher's dispositions towards mathematics at the elementary level (Newman, 1977).

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Appendix A – Signed Consent

SIGNED CONSENT FORM

Title of Research Study: An Analysis of Pre-Service Teachers' Understanding of Concepts Tested on the TIMSS Assessment

Researcher's Contact Information:
Monica Doriney
mdoriney@kennesaw.edu
912-220-1984

Research Supervisor:
Lynn Stallings
lstallin@kennesaw.edu
479-578-4477

Introduction
You are being invited to take part in a research study conducted by Monica Doriney of Kennesaw State University. Before you decide to participate in this study, you should read this form and ask questions about anything that you do not understand.

Description of Project
The purpose of this study is to investigate the mathematical knowledge of pre-service elementary teachers on key assessment items frequently missed by U.S. students on the 2011 Trends in International Mathematics and Science Study (TIMSS) assessment. The goal of this study is to investigate elementary pre-service teachers' understanding of mathematical concepts and skills in order to examine the possible impact on elementary students' performance. Research questions include the types of errors on the assessment items, the frequency of different sorts of errors, and a comparison of pre-service teacher and student performance on the items.

Explanation of Procedures
Pre-service teachers enrolled in course ECE 4401 Teaching Mathematics in Early Childhood Education at KSU will be invited to participate in this study. Participants will be asked for their age, gender, and race/ethnicity and will also complete a 32-item assessment composed of open-ended mathematics questions from the 2011 TIMSS assessment.

Time Required
Participants will be given the opportunity to participate during the fall or spring semester. Data collection and analysis may take place from September 2015 to May 2016.

Risks or Discomforts
There are no known risks associated with this research study; however, completing the assessment items may be an inconvenience.

Benefits

Participants may not directly benefit from this research. Your participation in the study may lead to better understanding of which mathematical concepts elementary pre-service teachers and their students misunderstand.

Confidentiality

The results of this participation will be confidential. The following procedures will be used to protect the confidentiality of your study records. I will keep all study records in a password protected electronic file. Research records will be labeled with a code. A master key that links names and codes will be maintained in a separate and secure password protected electronic file. The master key will be destroyed 3 years after the close of the study. All electronic files containing identifiable information will be password protected. Any computer hosting such files will also have password protection to prevent access by unauthorized users. I will only have access to the passwords. At the conclusion of this study, I may publish my findings.

Inclusion Criteria for Participation

Participants must be enrolled in ECE 4401 Teaching Mathematics in Early Childhood Education.

Signed Consent

I agree and give my consent to participate in this research project. I understand that participation is voluntary and that I may withdraw my consent at any time without penalty.

____

Signature of Participant or Authorized Representative, Date

____

Signature of Investigator, Date

PLEASE SIGN BOTH COPIES OF THIS FORM, KEEP ONE AND RETURN THE OTHER TO THE INVESTIGATOR

Research at Kennesaw State University that involves human participants is carried out under the oversight of an Institutional Review Board. Questions or problems regarding these activities should be addressed to the Institutional Review Board, Kennesaw State University, 1000 Chastain Road, #0112, Kennesaw, GA 30144-5591, (678) 797-2268.

Appendix B - Assessment

Race/Ethnicity ________________________ Gender ____________ AGE _______

An Assessment of Pre-Service Teachers' Understanding of Elementary Mathematics Concepts

PLEASE SHOW ALL WORK!!!!

1. Georgia wants to send letters to 12 of her friends. Half of the letters will need 1 page each and the other half will need 2 pages each. How many pages will be needed altogether?

2. Three thousand tickets for a basketball game are numbered 1 to 3,000. People with ticket numbers ending with 112 receive a prize. Write down all the prize-winning numbers.

3. 

A sequence of four figures is shown above.

If the figures were continued, how many circles would there be in Figure 10?

4.

Ingredients	
Eggs	4
Flour	8 cups
Milk	$\frac{1}{2}$ cup

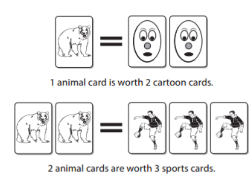
The above ingredients are used to make a recipe for 6 people. Sam wants to make this recipe for only 3 people.

What does Sam need to make the recipe for 3 people?

5. $3 + 8 = \square + 6$

What number goes in the box to make this number sentence true? Justify your answer.

6. The town fair had a booth where people could trade cards.



Some children went to the booth to trade cards.

Trading Animal Cards

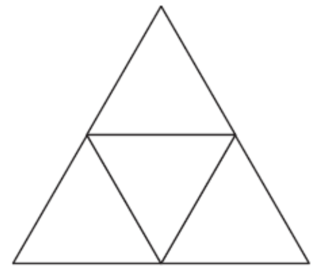
Jim had 8 animal cards to trade for sports cards. How many sports cards would he get?

7. The town fair had a booth where people could trade cards.  1 animal card is worth 2 cartoon cards.  2 animal cards are worth 3 sports cards. Some children went to the booth to trade cards. **Trading Animal Cards** Katrina had 6 animal cards. She wanted to trade them for as many cards as possible. How many cartoon cards would she get? ________ How many sports cards would she get? __________ Should she trade for cartoon cards or trade for sport cards?	8. Trading Sports Cards Steve had 15 sports cards to trade for animal cards. How many animal cards would he get? Brad had 8 cartoon cards to trade for sports cards. How many sports cards would he get?

9. Circle each number which is a factor of 12. Justify your answer. 1 2 3 4 5 6 7 8 9 10 11 12	10. Tom ate ½ of a cake, and Jane ate ¼ of the cake. How much of the cake did they eat altogether? Justify your answer.

11. In a soccer tournament, teams get: 3 points for a win 1 point for a tie 0 points for a loss Zedland has 11 points. What is the smallest number of games Zedland could have played?	12. Mary left Apton and rode at the same speed for 2 hours. She reached this sign.  Mary continues to ride at the same speed to Brandon. How many hours will it take her to ride from the sign to Brandon?

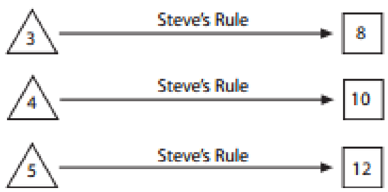
13. E stands for the number of pencils Pete had. Kim gave Pete 3 more pencils. How many pencils does Pete have now?	14. If the pattern 3, 6, 9, 12 was continued, what would be the 9^{th} number in the pattern?

15. Multiply the following expression. Round your answer to the nearest hundred. 9 x 22	16. Shade ½ of the large triangle. Justify your reasoning. 

17. Joan had 12 apples. She ate some apples, and there were 9 left. Write a number sentence to describe what happened?	18. Write these fractions in order from least to greatest. Explain your reasoning. $\frac{1}{2}, \frac{3}{5}, \frac{3}{10}, \frac{3}{8}, \frac{3}{6}$

19. A train left Redville at 8:45 a.m. It arrived in Bedford 2 hours and 18 minutes later. What time did it arrive in Bedford?	20. The scale on a map indicates that 1 centimeter on the map represents 4 kilometers on the land. The distance between two towns on the map is 8 centimeters. How many kilometers apart are the two towns?

21.



Steve used a rule to get the number in the □ from the number in the △. What was the rule?

22. Anna has these cards with numbers on them.

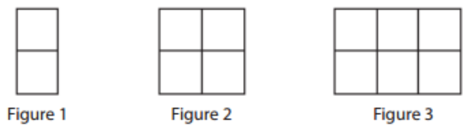
1	8	6	5	2

What is the smallest three-digit number she can show with the cards? She may use each card only once. Justify your answer.

23. Paint comes in 5 liter cans. Sean needs 37 liters of paint. How many cans must he buy? Justify your reasoning.

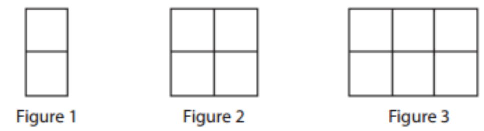
24. Duncan first traveled 4.8 km in a car and then he traveled 1.5 km in a bus. How far did Duncan travel?

25. Bill is arranging squares in the following way:



Draw Figure 5.

26. Bill is arranging squares in the following way:



How many squares would Bill need to make Figure 16?

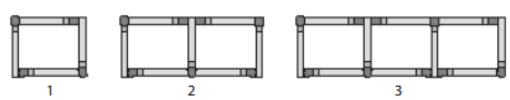
27. $23 \times 19 =$

28. Cooney has to form figures 1 to 4 with matches.

Figures 1, 2, and 3 are shown below.

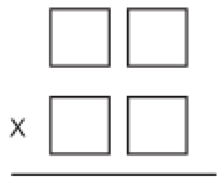
He needs four matches to form figure 1, seven matches to form figure 2, and ten matches to form figure 3.

He uses the same rule each time to make the next figure in the pattern.



How many matches will he need to form figure 4?

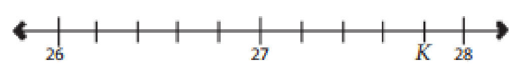
29. Place the four digits 3, 5, 7, and 9 into the boxes below in the positions that would give the greatest result when the two numbers are multiplied.



30. Kim is packing eggs into boxes. Each box holds 6 eggs. She has 94 eggs. What is the smallest number of boxes she needs to pack all the eggs?

31. Six hundred books have to be packed into boxes that hold 15 books each. Write an expression that could be used to find the number of boxes needed. Explain your reasoning.

32. What number does K represent on this number line? Justify your reasoning.



Appendix C

Tables, Figures, and Templates

Table 7

Question topic and Cognitive Domain

Assessment Question Number	Main Topic	Cognitive Domain
1	Whole Numbers	Applying
2	Whole Numbers	Reasoning
3	Patterns and Relationships	Reasoning
4	Whole Numbers	Applying
5	Number Sentence with Whole Numbers	Knowing
6	Whole Numbers	Reasoning
7	Whole Numbers	Reasoning
8	Whole Numbers	Reasoning
9	Whole Numbers	Knowing
10	Fractions and Decimals	Knowing
11	Whole Numbers	Reasoning
12	Whole Numbers	Reasoning
13	Number Sentence with Whole Numbers	Applying
14	Patterns and Relationships	Applying
15	Whole Numbers	Knowing
16	Fractions and Decimals	Applying
17	Number Sentence with Whole Numbers	Applying
18	Fractions and Decimals	Knowing
19	Whole Numbers	Applying
20	Whole Numbers	Reasoning
21	Patterns and Relationships	Applying
22	Whole Numbers	Knowing
23	Whole Numbers	Applying
24	Fractions and Decimals	Applying
25	Patterns and Relationships	Applying
26	Patterns and Relationships	Reasoning
27	Whole Numbers	Knowing
28	Patterns and Relationships	Applying
29	Whole Numbers	Reasoning
30	Whole Numbers	Applying
31	Whole Numbers	Applying
32	Fractions and Decimals	Knowing

Table 8

Percentage of Questions at Each Cognitive Domain

Cognitive Domain	Percentage
Knowing	25%
Applying	44%
Reasoning	31%

Table 9
Percentage of Questions for Each Topic

Main Topic	Percentage
Whole Number	56%
Number Sentence with Whole Numbers	9%
Patterns and Relationships	19%
Fractions and Decimals	17%

Table 10
Initial Error Type Analysis by Questions for Each Participant Form

Question # _______		Type of Error					
Participant	Correct	Number Selection	Missing Step	Operational	Computational	Random	Omission
1							
2							
. . .							
n							

Table 11
Initial Error Type Analysis by Participant for Each Question Form

Participant # ______		Type of Error					
Question	Correct	Number Selection	Missing Step	Operational	Computational	Random	Omission
1							
2							
. . .							
n							

Table 12

Consolidated Error Type Percentages

Question	% Incorrect	% Number Selection	% Missing Step	% Operational	% Computational	% Random	% Omission
1							
2							
. . .							
32							

Table 13

Error Type by Cognitive Domain

	Cognitive Domain		
Error Type	Knowing	Applying	Reasoning
Number Selection			
Missing Step			
Operational			
Computational			
Random			
Omission			

Table 14

Error Type by Main Topic

		Main Topic			
Error Type		Whole Numbers	Patterns and Relationships	Number Sentences with Whole Numbers	Fractions and Decimals
	Number Selection				
	Missing Step				
	Operational				
	Computational				

	Random				
	Omission				

Table 15

Comparison of US Student Scores with Sample of Pre-Service Teachers

	Question Number	% Correct Student	% Correct Pre-Service Teacher
Knowing	5	47%	76%
	9	46%	71%
	10	35%	69%
	15	68%	62%
	18	62%	60%
	22	60%	80%
	27	59%	93%
	32	65%	95%
Applying	1	47%	73%
	4	33%	76%
	13	83%	55%
	14	83%	67%
	16	75%	71%
	17	92%	89%
	19	57%	89%
	21	63%	38%
	23	53%	71%
	24	74%	85%
	25	63%	82%
	28	58%	75%
	30	56%	56%
	31	63%	65%
Reasoning	2	31%	80%
	3	47%	75%
	6	34%	73%
	7	25%	76%
	8	21%	62%
	11	34%	67%
	12	33%	60%
	20	59%	85%
	26	59%	76%
	29	38%	27%

Appendix D – **Raw Data, Graphs and Charts**

Table 16

Consolidated Error Type Percentage

Question	Correct	Incorrect	Number Selection Error	Missing Step Error	Operational Error	Computational Error	Random Error	Omission
1	40	15	0	1	0	4	0	8
2	44	11	0	3	0	2	0	6
3	41	14	0	1	1	2	0	10
4	42	13	0	3	0	2	0	8
5	42	13	0	0	0	2	0	11
6	40	15	0	0	0	3	0	12
7	42	13	0	0	0	2	1	10
8	34	21	0	2	2	4	0	13
9	39	16	0	2	1	1	0	12
10	38	17	0	0	1	2	0	14
11	37	18	0	0	0	2	1	15
12	33	22	0	1	0	3	0	18
13	30	25	0	0	10	0	0	15
14	37	18	0	0	0	2	0	16
15	34	21	0	4	0	1	0	16
16	39	16	0	0	0	0	0	16
17	49	6	0	2	2	0	0	2
18	33	22	0	7	0	8	4	3
19	49	6	0	0	1	2	1	2
20	47	8	0	0	3	0	0	5
21	21	34	0	15	6	5	2	6
22	44	11	0	1	0	0	0	10
23	39	16	0	7	0	2	0	7
24	47	8	1	0	0	1	0	7
25	45	10	0	1	0	0	0	8
26	42	13	0	3	0	2	0	8
27	51	4	1	0	0	4	0	0
28	41	14	0	2	0	1	0	11
29	15	40	0	0	1	23	2	13
30	31	24	0	0	3	4	2	15
31	36	19	0	1	0	0	1	17
32	52	3	2	0	0	1	0	2

Table 17

Error Type for each Student Per questions

	Question Numbers																															
Student Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM			C		C		C				OM	OM	OM	OM	OM	OM
2	OP			OM	OP	OM		OM					OM	OM	OM	OM				MS	OM	C		OM		OM		OM				
3							OM	OM	OM	OM	OM	OM	OM	OM	OM	OM			OM													
4		NS			C								C																			
5		R			OP					MS			C			C	N		OP													
6					C	MS							C																			
7		R			R						C		C											C	MS						MS	
8					OP		MS					C	C						OM					C				OM				
9		C	R	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM												C				
10											C		C	R	OM	OM	C															
11					OP								C															OM				
12		C		OP						OM	OM	OM	OM	OM	OM	OM																
13		OM											C																			
14	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM											OM		
15				OP	C		MS		OM	OM	OM	OM	OM	OM	OM	OM		MS		MS							C				C	
16		R			OM					OP			C																			
17		C		OP	C						C	OM	OM	OM	OM	OM				MS				C							MS	
18	OP	R			OP					C				C			C	C			C		R		OP	OM		MS	OP			
19					OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	N							OP		OP	C	OM				
20					MS								R																			
21					MS					MS			R	MS			C															
22									MS				N	MS										MS	OM		R	OM	OM	OM	OM	OM
23																										OM	OM	OM	OM	OM	OM	OM
24										MS			C				OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM
25																																
26			OP		MS		MS							C																C		
27					MS	OM	MS						C	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM
28		MS	C	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM															MS	
29					MS												MS		MS					MS								
30																								OP								
31		MS			MS		MS								OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM
32													C		MS						OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM
33		MS			MS	OM						OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM
34		MS			MS								OP											OM	OM	OM	OM	OM	OM	OM	OM	OM
35		MS			OP					C		MS	C			OM			C	C		C		C		C	OM	OM	OM	OM	OM	OM
36		MS			C		C	C					C			OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM
37							MS						C		OM	OM			OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM
38																												C			OM	OM
39		NS	C						N													OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM
40							MS				C		C	MS	R		OM			C	OM	OM	C	OM			OM	C			MS	
41													C			OM				OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM
42	MS	C			MS		C					MS	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM
43					MS	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM	OM																
44			X	X			X	X			X	X		C	X	X	C	C	X	X		C	X	X			X	X			X	X
45	MS	C	X	X	MS		X	X			X	X	C		X	X			X	X			X	X			X	X		C	X	X
46		C	X	X			X	X			X	X	C		X	X			X	X			X	X	MS	C	X	X			X	X
47			X	X	MS		X	X			X	X	C	R	X	X			X	X			X	X			X	X			X	X
48			X	X	MS		X	X			X	X			X	X			X	X	C		X	X	C		X	X			X	X
49		C	X	X	MS		X	X			X	X		C	X	X			X	X			X	X			X	X			X	X
50			X	X	R		X	X			X	X	C		X	X			X	X			X	X			X	X			X	X
51		C	X	X	C		X	X			X	X	C		X	X		MS	X	X			X	X			X	X			X	X
52			X	X			X	X			X	X	C		X	X			X	X			X	X			X	X			X	X
53		MS	X	X		OM	X	X			X	X	OM	OM	X	X		MS	X	X			X	X			X	X	OM		X	X
54			X	X			X	X			X	X							X	X			X	X			X	X			X	X
55			X	X	MS		X	X	C		X	X			X	X			X	X			X	X			X	X			X	X

Figure 23

Number of Correct and Incorrect Responses per Question

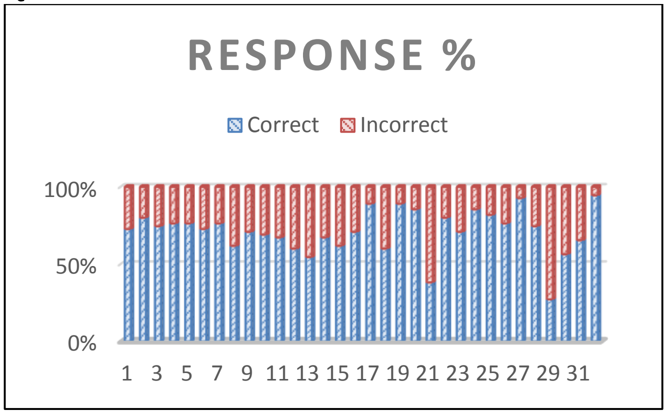


Figure 24

Overall Percentage of Correct and Incorrect Responses

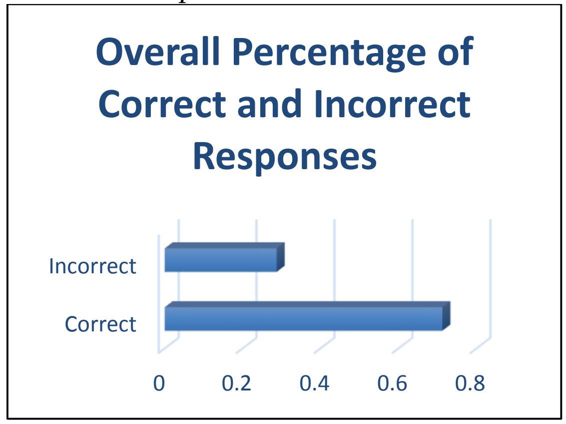


Table 18

Cohen's Kappa Statistics

Response	Kappa	SE Kappa	Z	P(vs > 0)
1	0.532940	0.0696407	7.6527	0.0000
2	0.885466	0.0725418	12.2063	0.0000
3	0.922905	0.0725476	12.7214	0.0000
4	0.871694	0.0724812	12.0265	0.0000
5	0.927647	0.0723575	12.8203	0.0000
Overall	0.878019	0.0445017	19.7300	0.0000

Figure 25

Percentage of Correct Answers Provided by Pre-Service Teachers on the TIMSS Assessment Items